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PERT CPM



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PROJECT

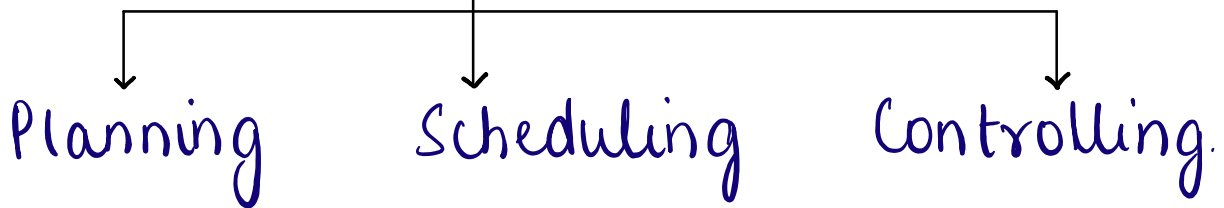


- Any work which we do is termed as Project.
- To execute the project, resources are required.
Ex : Manpower, machine (equipment), money, material, time.
- Engg. deals with technology where as project management deals with synchronisation of the technology with manpower.
- Every project has 3 - Basic objective.
 1. Project should be completed in least time.
 2. Project should be completed in least cost.
 3. Project should be completed by optimum use of resources that are available.

ELEMENTS OF PROJECT MANAGEMENT



- There are 3 basic elements of project management.



1. Planning

It is the process of defining objective of project and to identify resources required for its completion.

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2. Scheduling

It is the process of arranging the objective define during planning in sequence and allocation of resources to these activities.

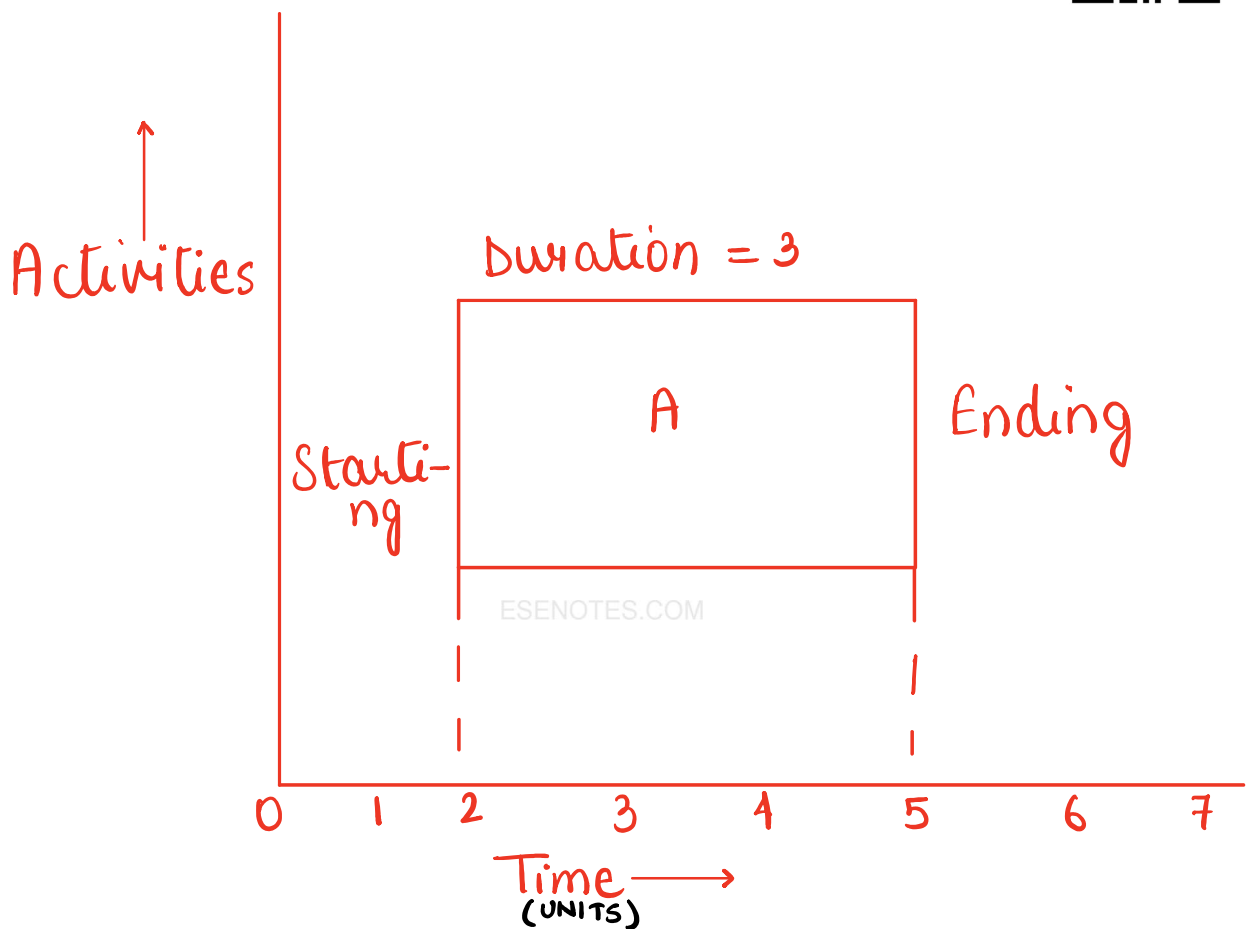
3. Controlling

It involves execution of planning and scheduling.

- If there is any deviation from original plan it also includes rescheduling.

METHODS OF PROJECT MANAGEMENT

1. Bar chart / Gantt chart



Bar chart is graphical representation of activity vs time.

- Here time durations are marked on x-axis &

Activities are marked on y-axis.

- Length of bar represents duration of activity.

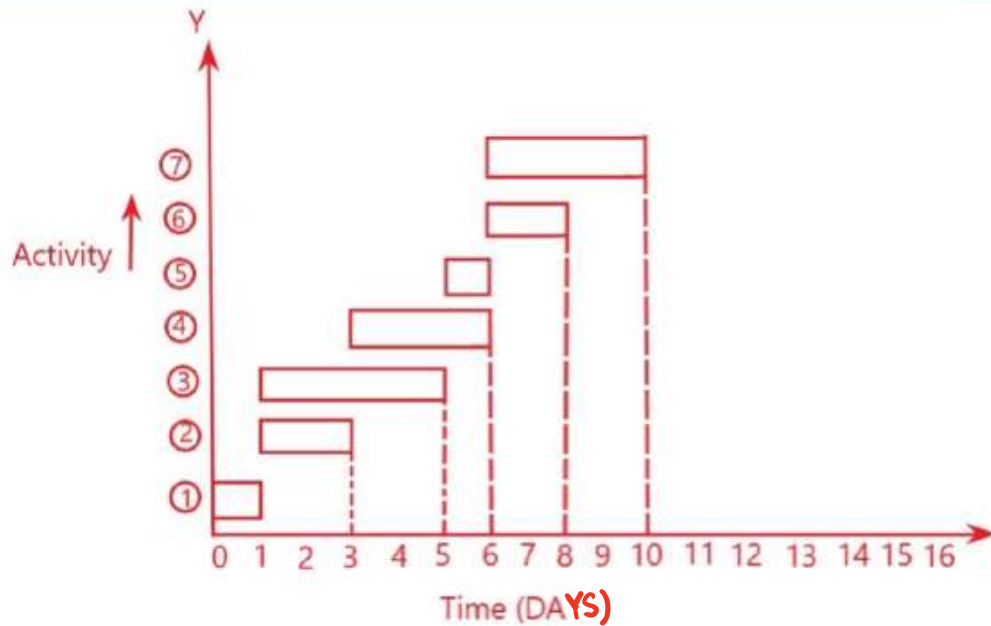
- Activities are shown with the help of bar.
- Start point of bar represents starting time of activity and end of bar represents finish time of activity.



Question : Draw a bar chart for certain project

Activity(No.)	1	2	3	4	5	6	7
Duration(days)	1	2	4	3	1	2	4

- Activities 2 and 3 can be done concurrently and both must follow Activity 1.
- Activity 2 must precedes activity 4.
- Activity 5 can not begin until both activity 2 and 3 are completed.
- Activity 6 and 7 can be started only after activity 4 and 5 completed.
- Find the project completion time.



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Advantages of bar chart.

1. It is simple to draw and understand.
2. No skilled manpower is required.
3. It provides visual representation of the project, where starting and finishing can be observed clearly.

Limitations of bar chart.

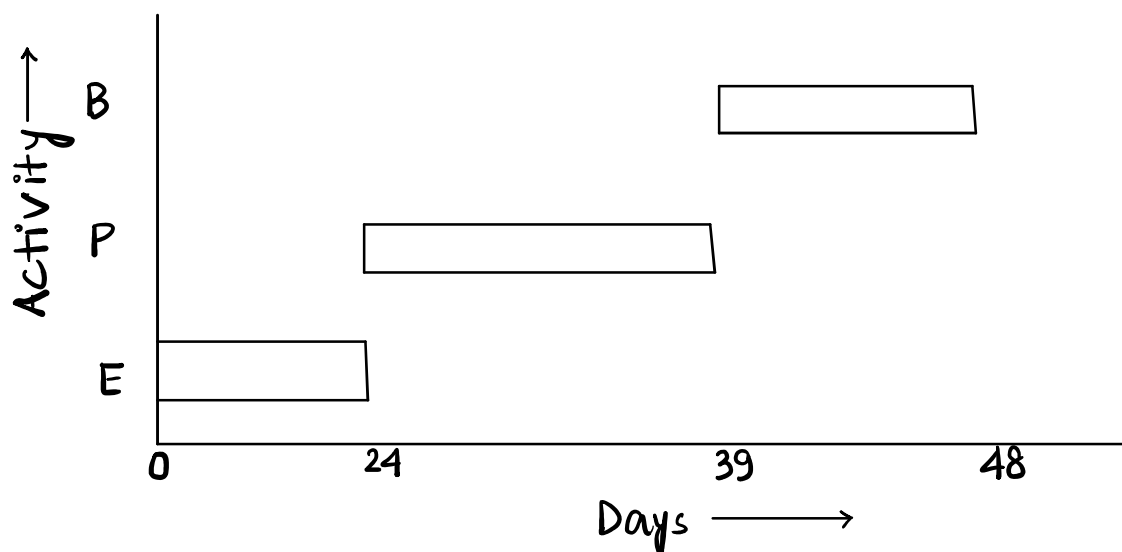


- (i). Lacks degree of details
- (ii). Review of project progress. Hence, it is not used as tool for controlling the project. This drawback of bar chart eliminated by hatching the half width of the bar upto the extent it represent project progress.

(iii) Activity Inter-relationship

Example: Project- laying of Pipeline

- a) Excavation (24 days)
- b) Placing the pipe line (15 days)
- c) Backfilling (9 days)



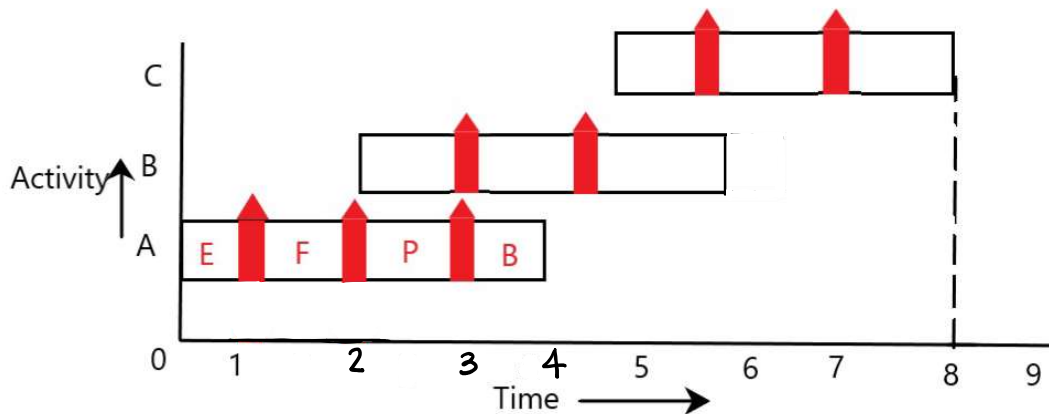
(iv) Time Uncertainty.



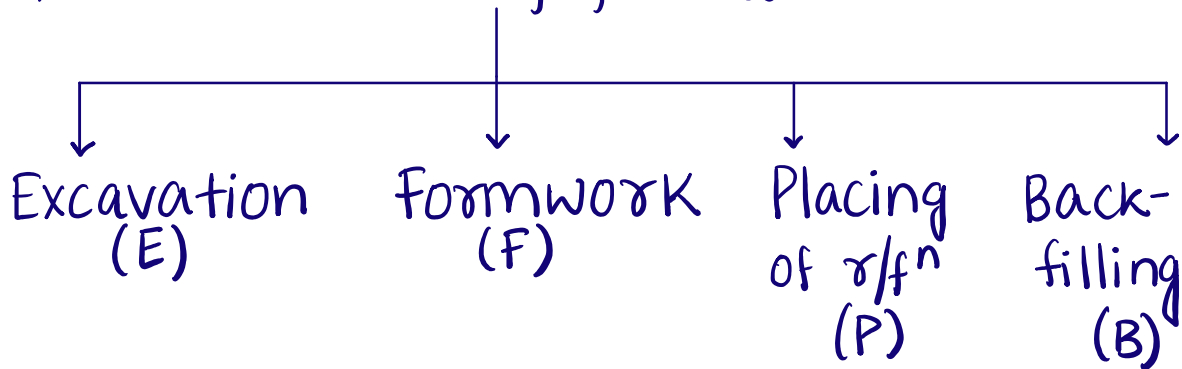
(v) No cost optimisation.

2. Mile Stone chart.

- It is an improvement of bar chart. In this case certain portion of the activity, that is completed in terms of mile stone.
- Milestone can be shown or drawn with the help of arrow, circle or square.
- Each milestone can be considered as an event of main activity.
- Here interrelationship between sub activities of a particular activity is established but different activities still not established.



A = construction of foundation.



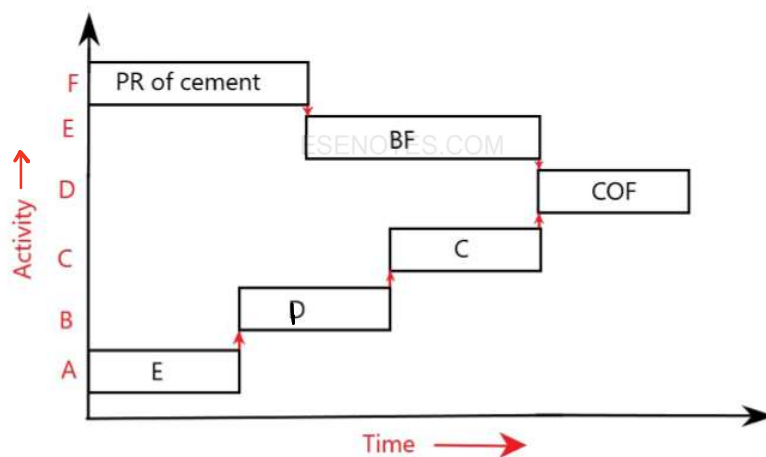
NOTE:

Hence mile stone chart is event oriented and bar chart is activity oriented.



3. Linked bar chart.

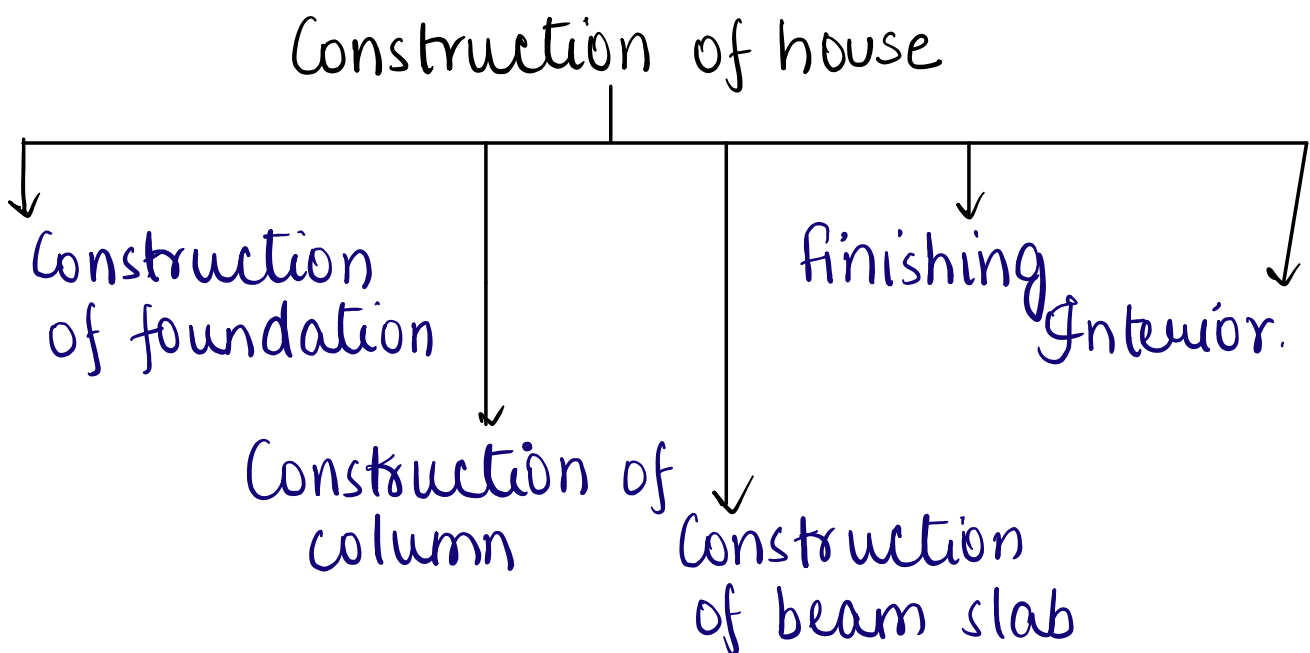
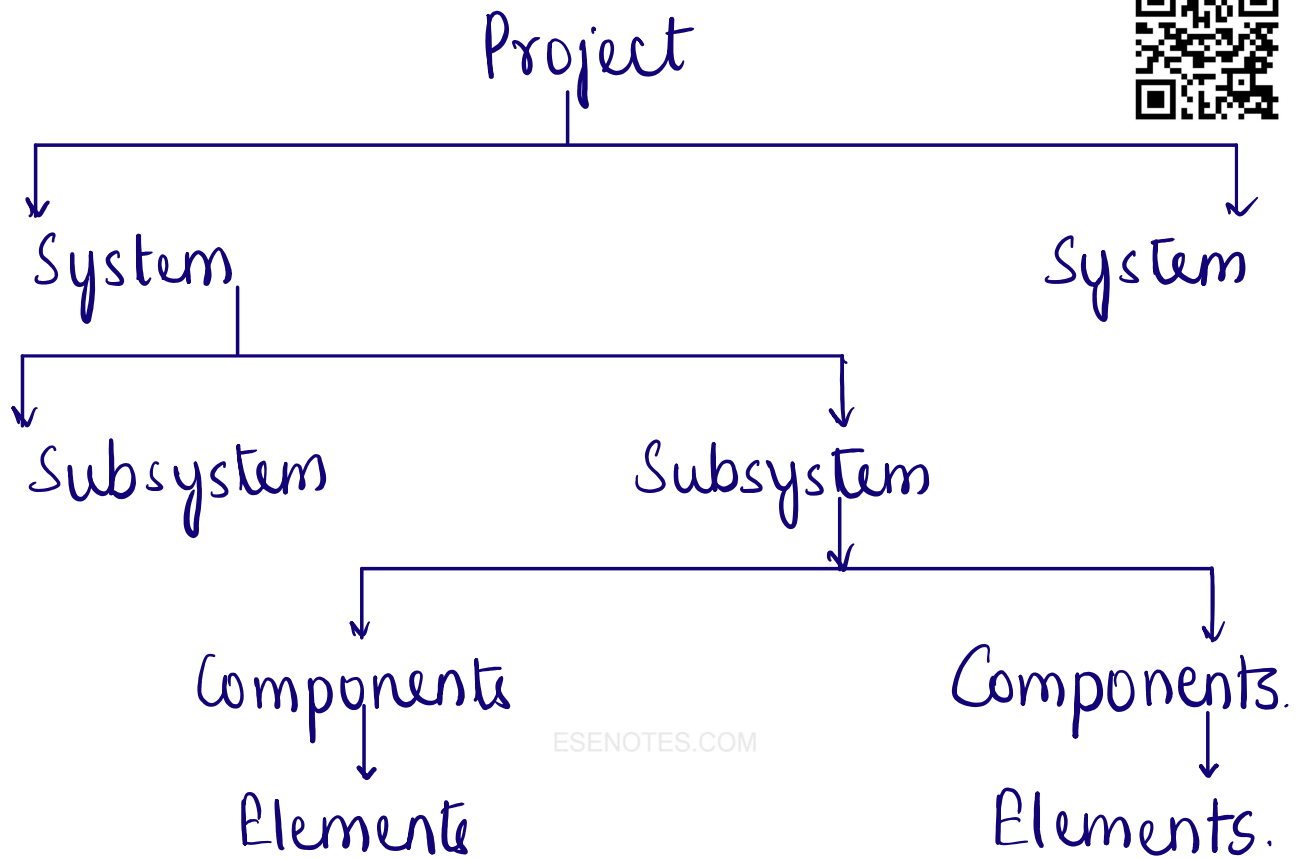
It is the modification of bar chart and milestone chart, in which interrelationship between activities is shown with the help of links.

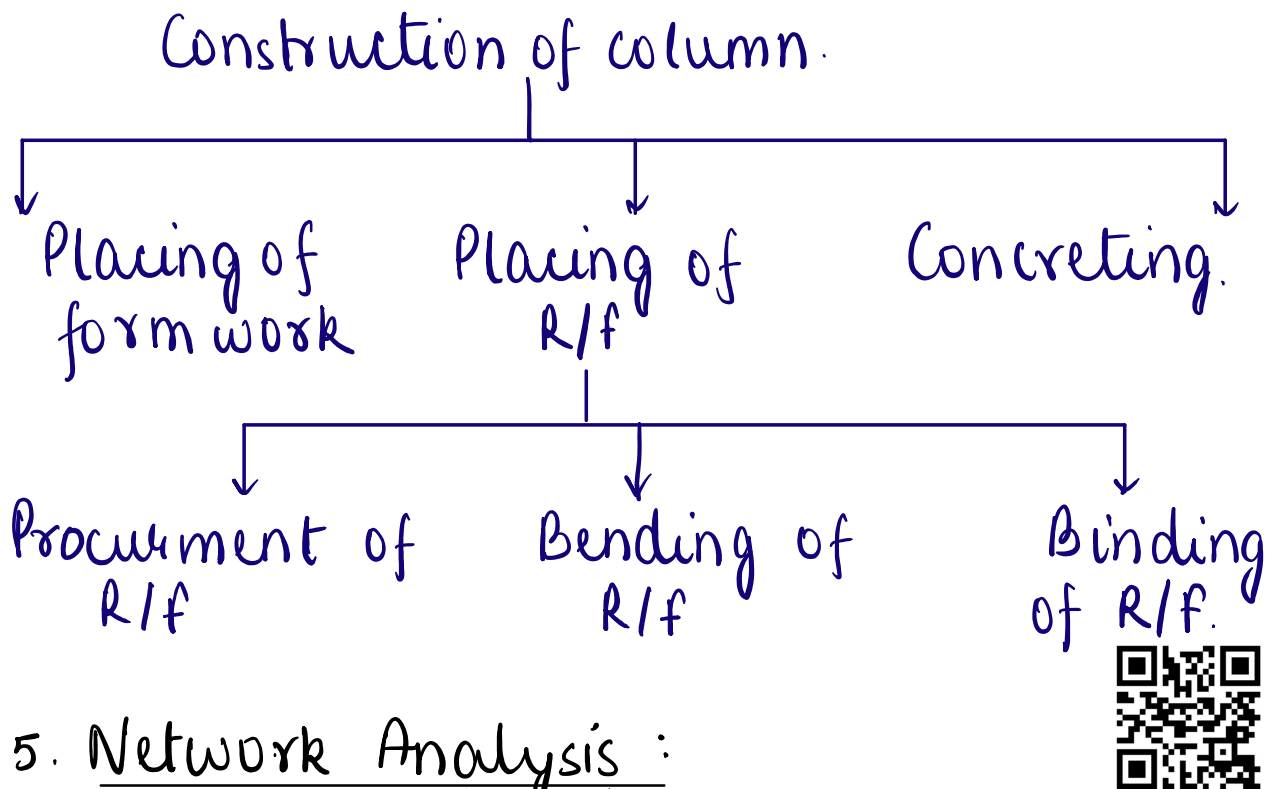


4. Work Breakdown Structure

It is the process of breaking the entire project to be executed into no. of system, subsystem components and its elements.

so as to simplify the execution of project.





5. Network Analysis :

It is graphical and logical representation of diff activities.

- It is modification of bar chart, mile stone, linked chart where all the draw back eliminate.
- Network are of two type:
 - a) Activity on arrow (AOA) / Arrow diagram.
 - b) Activity on node (AON) / Precedence.

Fundamentals of Network

Network consist of different activities connected logically and sequentially such as required for completion of project.



1. Activity on arrow network diagram
It consist of activities being mark on arrows and event mark on node.

Activity : Actual performance of the task is termed as activity.

Ex : Construction of foundation, concreting preparing for ESE, creating nuisance in class.

- for completion of an activity resources are required.
- Here activity is denoted by an arrow.
- Tail of arrow signifies start of activity and head of arrow signifies completion of an activity.
- Length and direction of arrow has no significance. But generally they are

drawn from left to right.

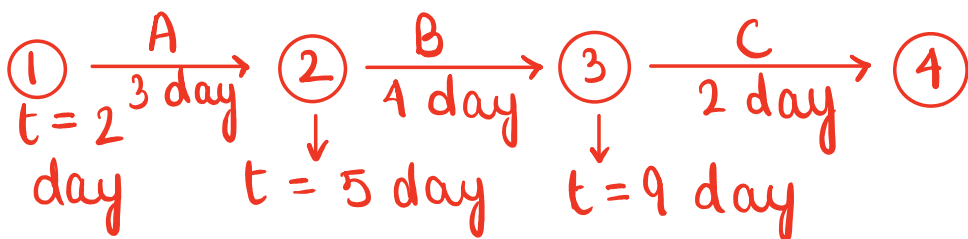


→ Description { A, B, C
 } tail or head event
 Name

Types of activity

1. Serial Activity

An activity which can be started only after the completion of another activity is termed a serial activity.

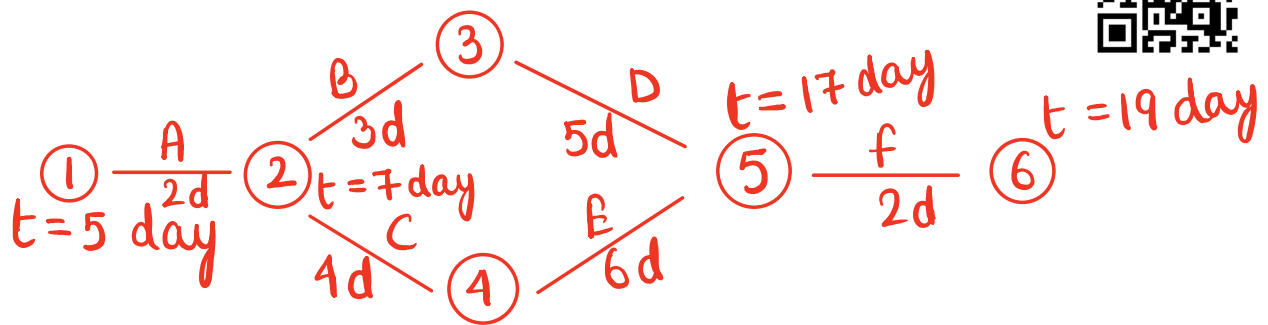


Here activity A, B, C are serial activity.

2. Parallel Activity

Activities which can be carried out simultaneously are termed as Parallel

activity.



- Series activity A B D f, A C E f
- Parallel activity B || C, D || C, D || E.

Parallel activities are independent to each other.

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Event: An event is an instant of time which marks the start or completion of an activity.

- occurrence of an event do not required any resources.

Example:

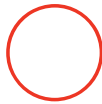
- Construction of foundation started.
- Construction of foundation completed.
- Concreting started.
- Concreting finished.

- Preparation of ESE examination started.

- Preparation of ESE examination completed.



- Here event are marked on nodes and are represented by circle, square, rectangular.



Types of an event

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1. Tail event

An event which marks the starting of an activity.

A tail event which marks the starting of project is termed as initial event.

- Except last event all events of network are tail event.

- First tail event is initial event.

2. Head event.

An event which marks the completion of an activity is termed as head event. A head event which marks the completion of project is termed finish event/ End event.

- Last head event is termed as finishing event.



3. Dual Role event

Event which acts as both for different activities is termed as dual Role event. Except initial and end event all the events are dual role event.

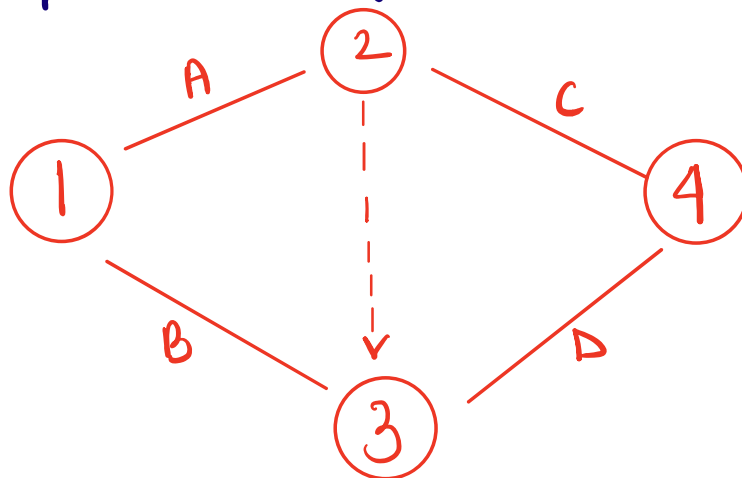


NOTE :

Dummy : It is virtual, artificial, superficial activity drawn in network only (it is not the part of project) to establish relationship between other activities.

- Since this activity is not the part of actual project, it consumes no resources.

- It is represented by a broken arrow.



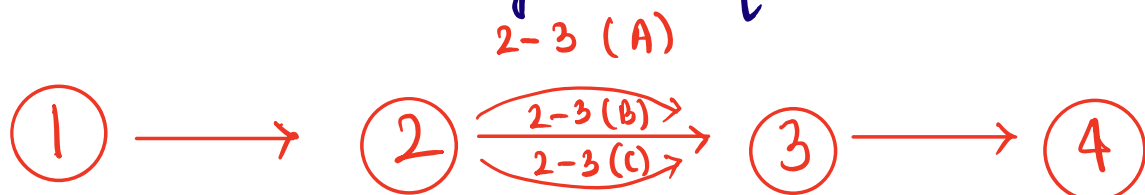
Here dummy activity 2-3 indicates that Activity D cannot be started until activity A is finished.

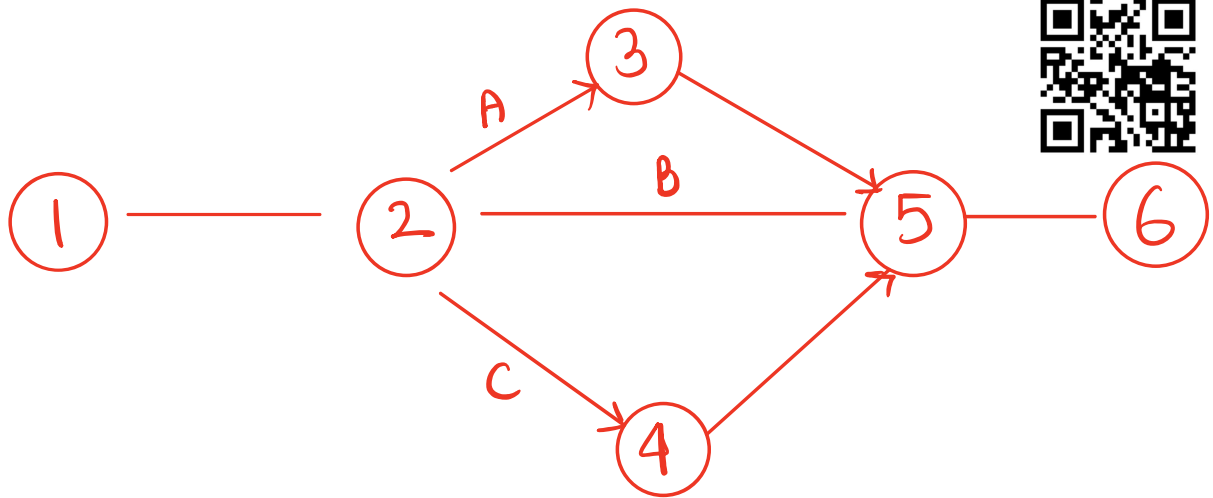
Dummy activity helps in establishing the following relationship.

1. Grammatical Relation :

Here Dummy is used to establish the uniqueness of identity.

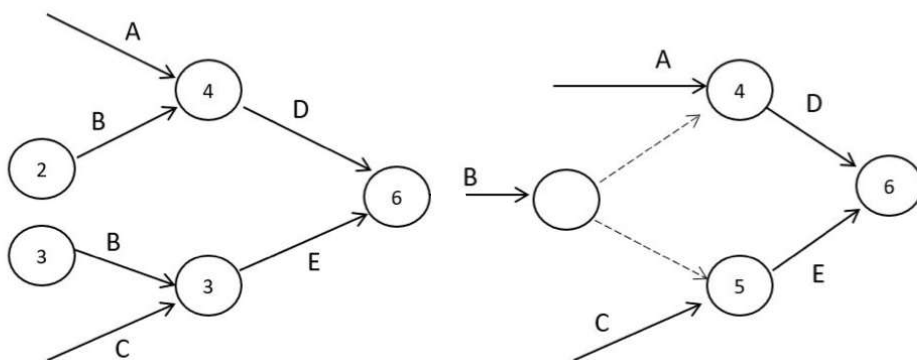
- If more than one activity starts from some node and terminates also at some other node dummy is required.





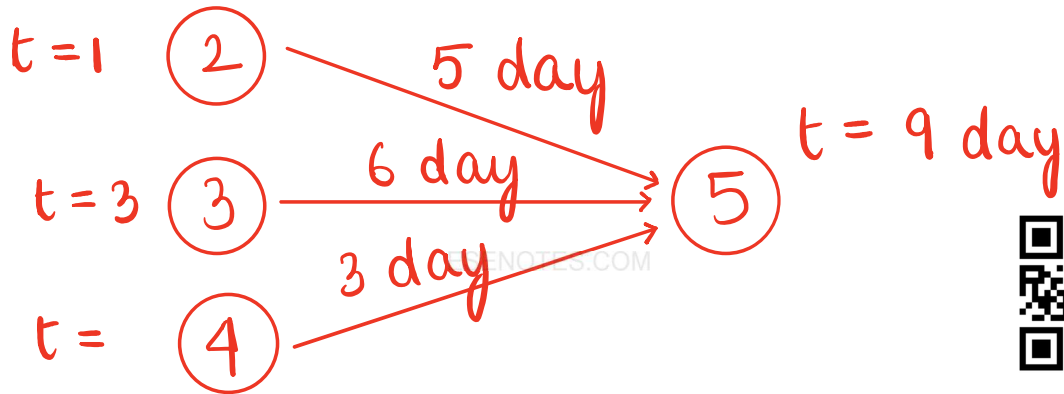
Logical Relationship

Dummy is used to establish the logical relation in the network, when an activity is common to two set of successor activity in such case Dummy establishes uniqueness of activity.

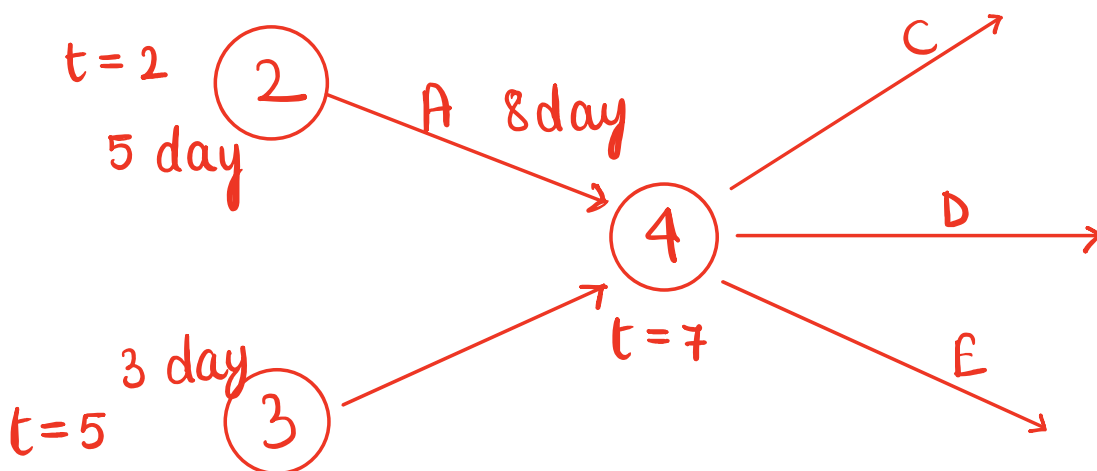


Rules to draw network

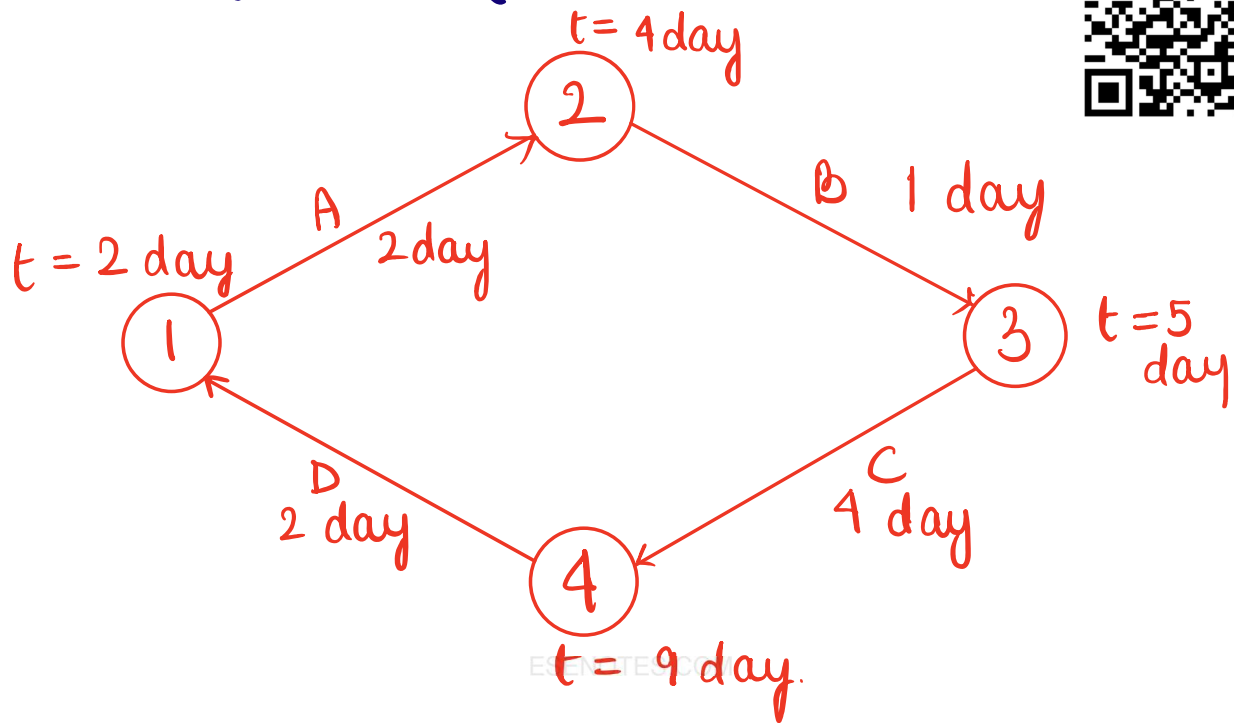
1. A network will have only one initial event and one finish end (initial event has only outgoing arrow, finish event has only incoming arrows).
2. An event cannot occur until all its incoming activities are completed.



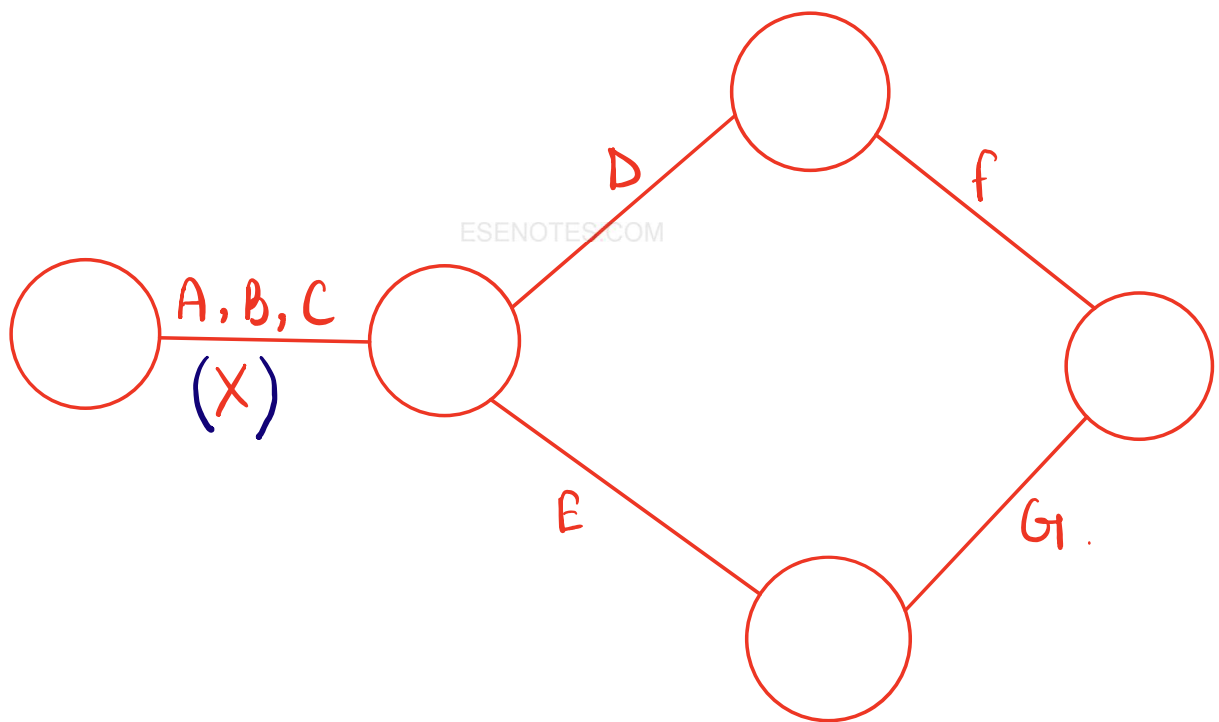
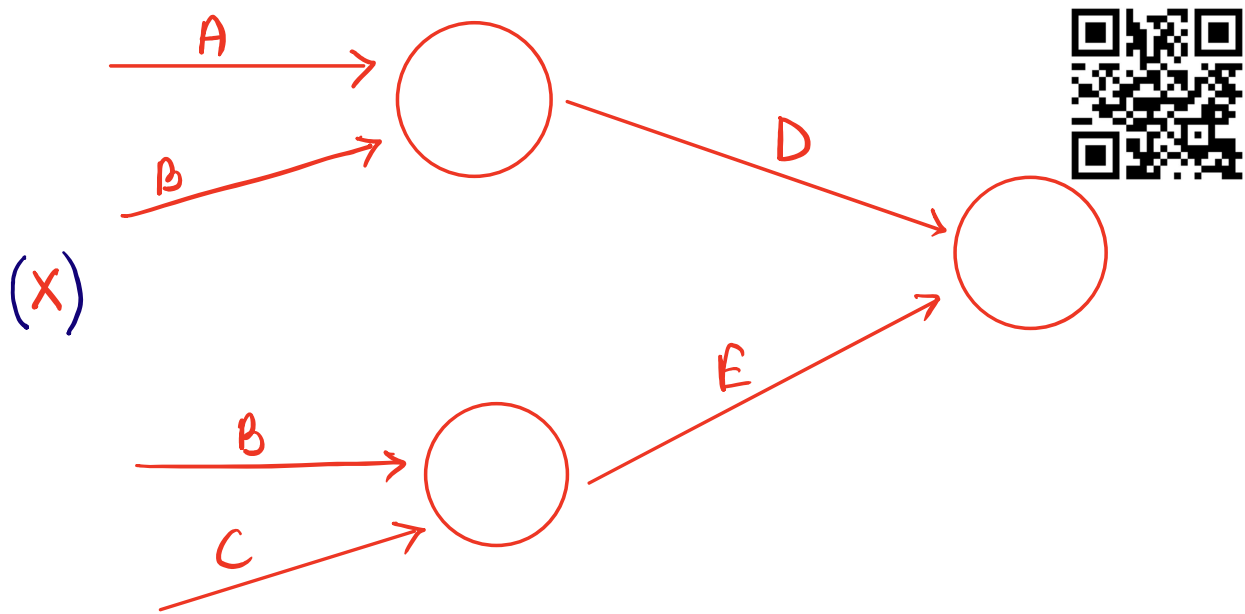
3. No activity can start until its tail event occurs.



4. No event can occur more than once, hence there should be no looping error in the network.



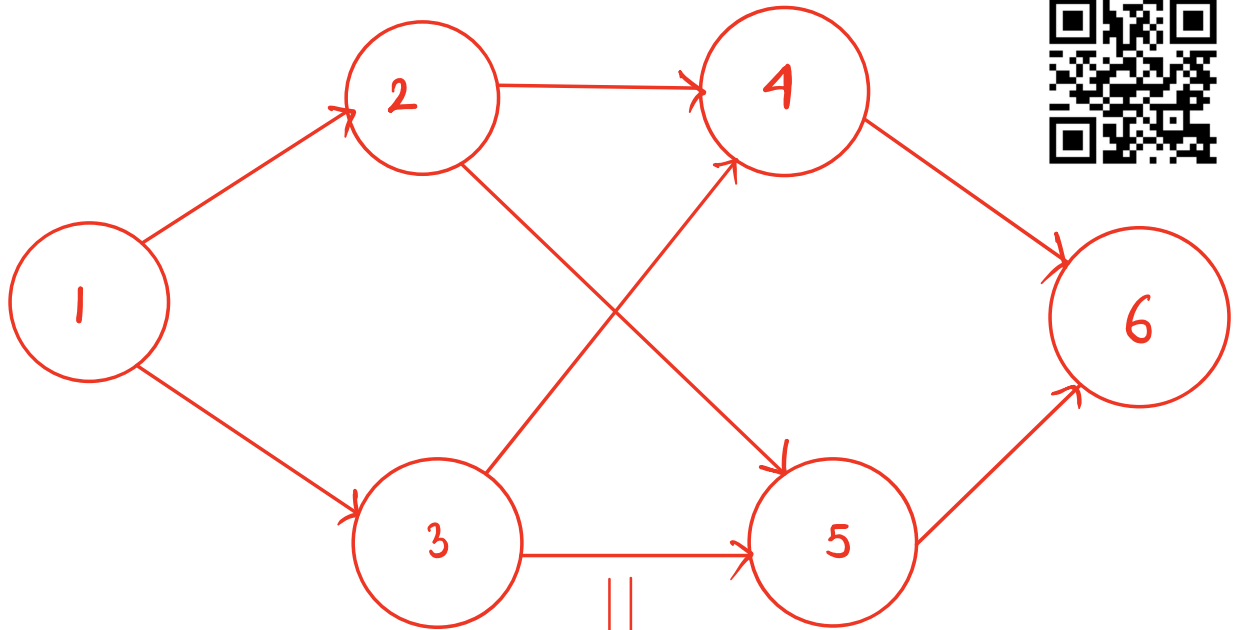
5. An arrow should represent only one activity i.e., two or more arrows should not be used to represent same activity.
6. Similarly two or more activity should not be represented by single arrow.



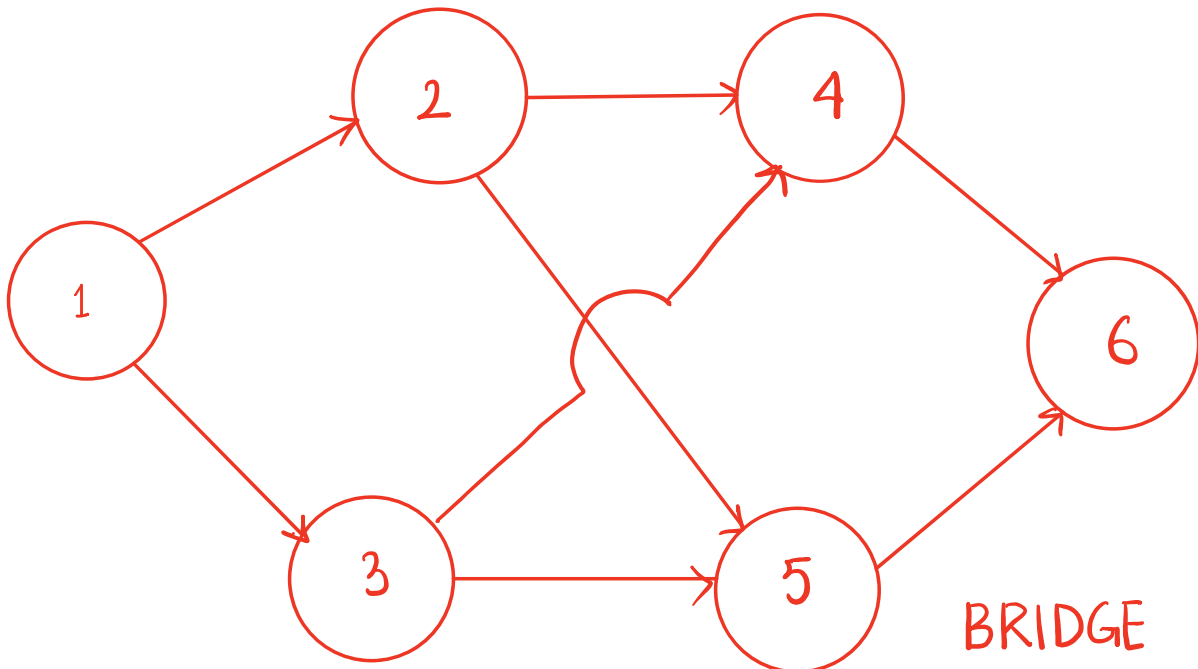
7. As far as possible curved arrow must be avoided.

8. As far as possible crossing of arrows

must be avoided and if it cannot avoided bridging of activity must be done.



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Time flow is from left to right.

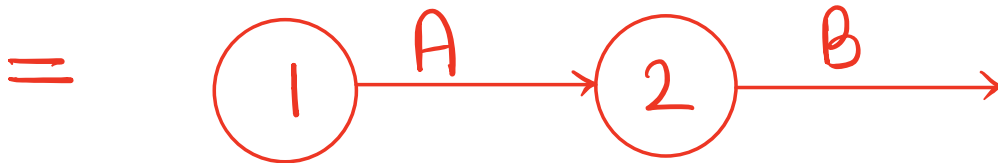
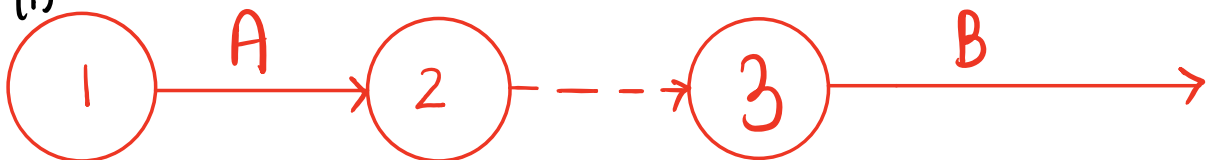
NOTE :

for development of network, dummies are required to establish logical and grammatical relationship. But it is difficult to identify where dummies must be provided. Hence, to overcome dummies are provided in conservative way which results, redundant / unwanted dummies; that removed by following ways.

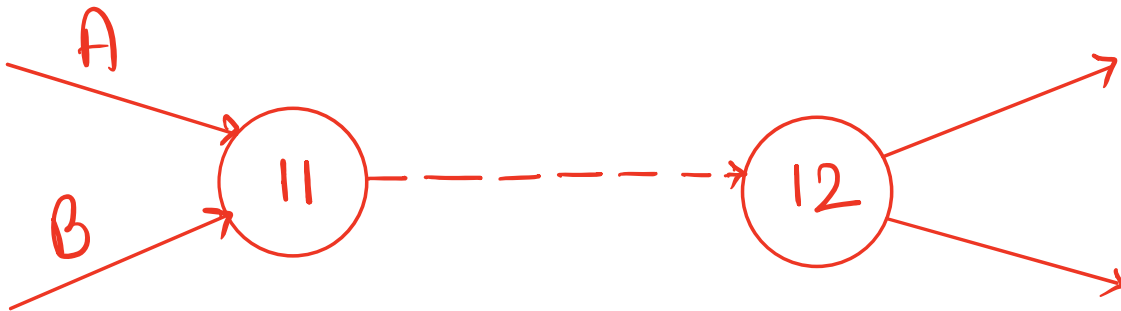


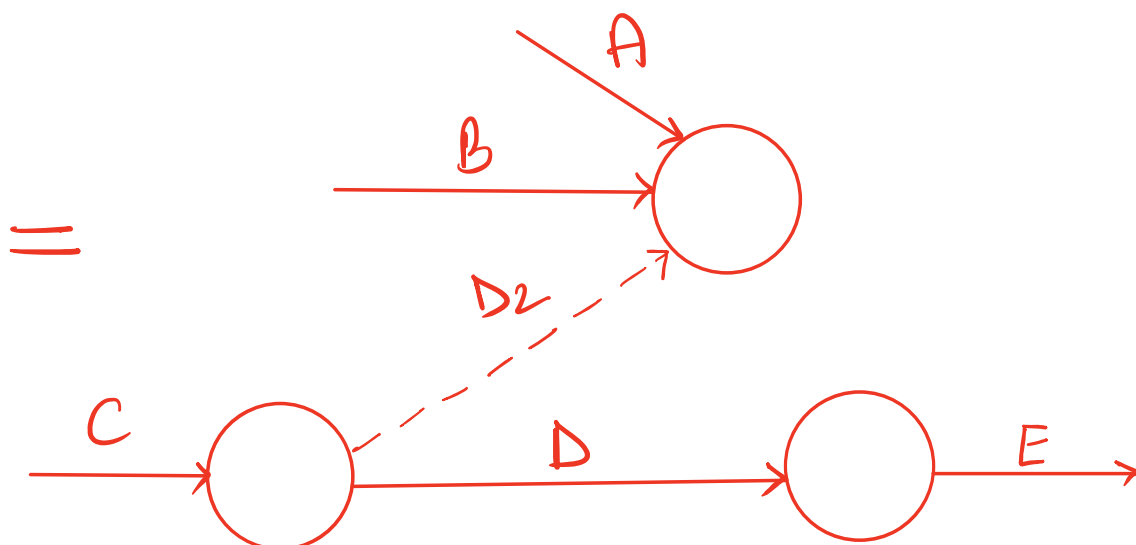
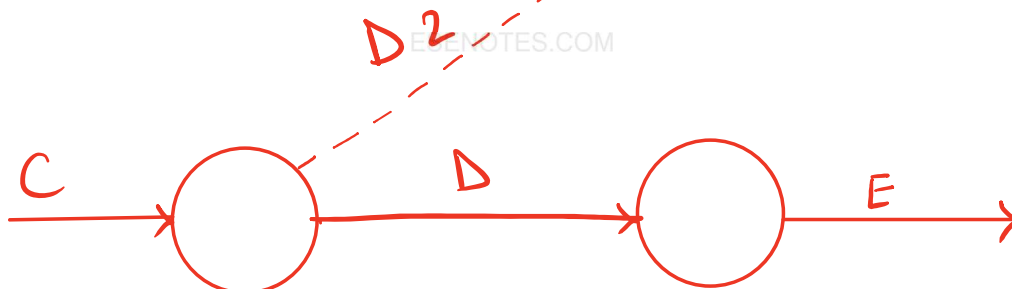
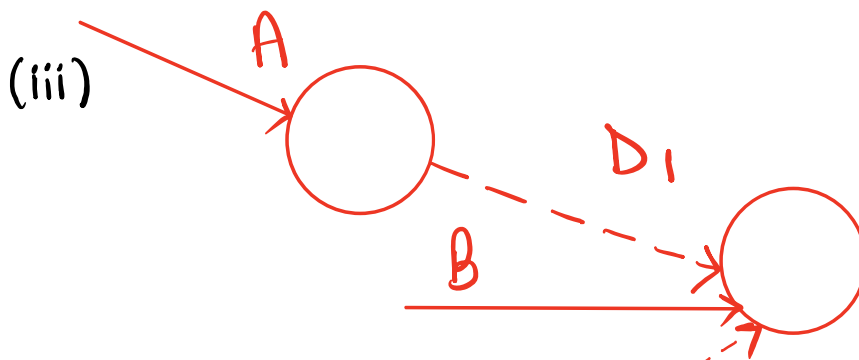
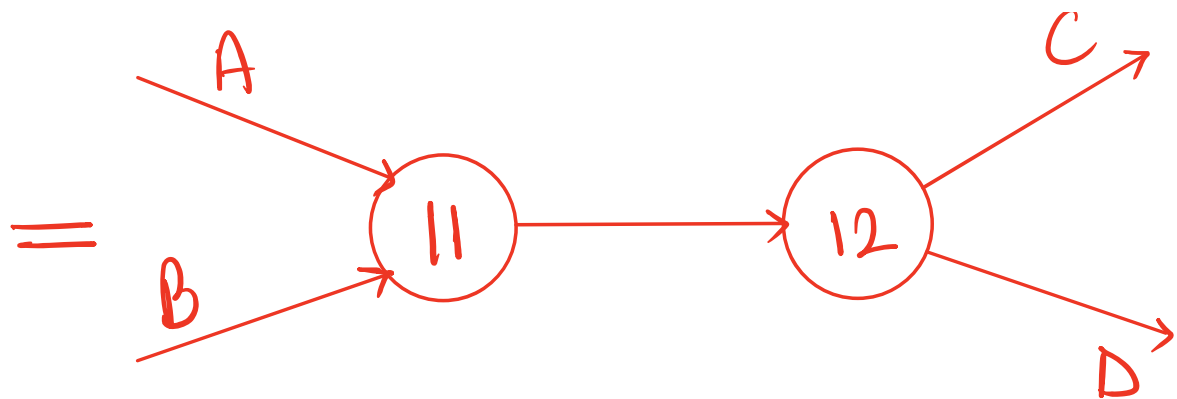
1. If dummy is the only activity emanating from its tail event it can be removed.

(i)

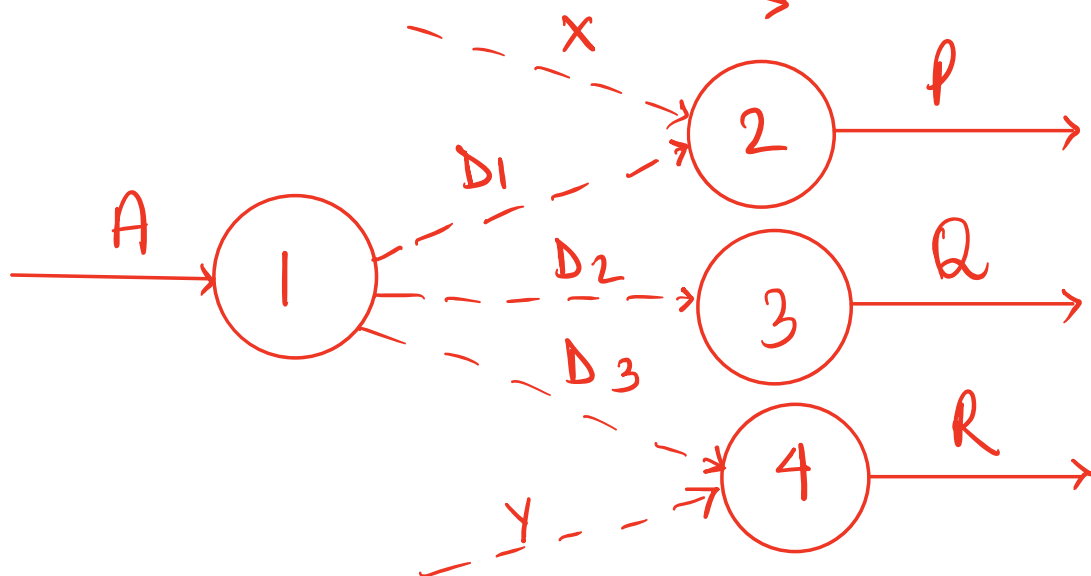
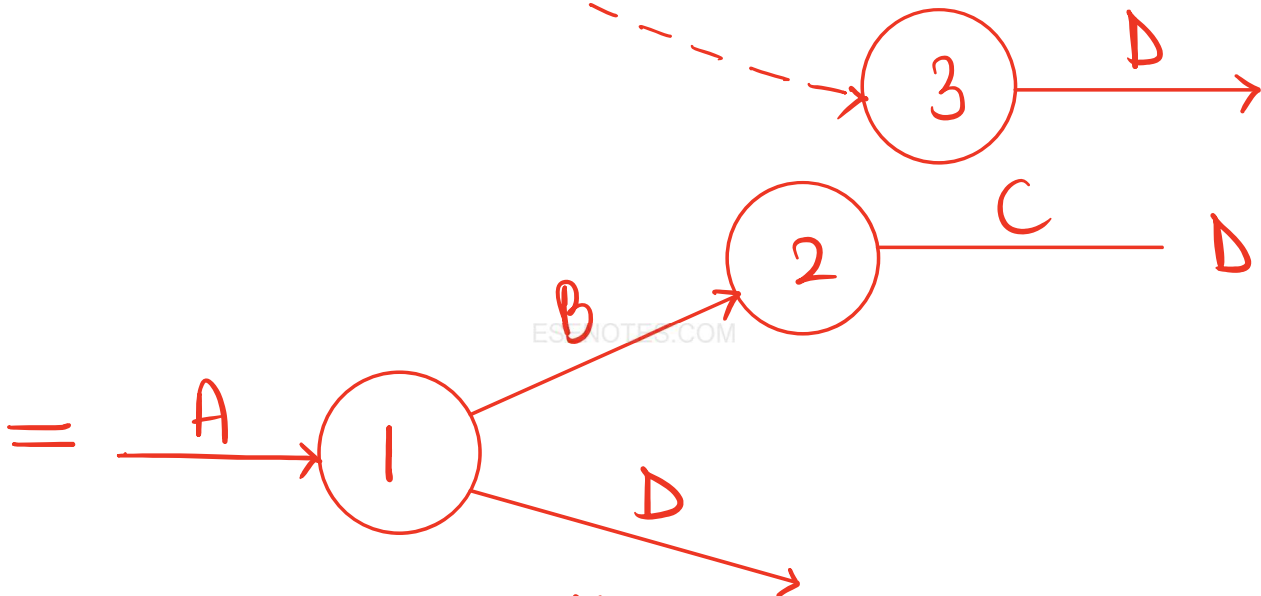
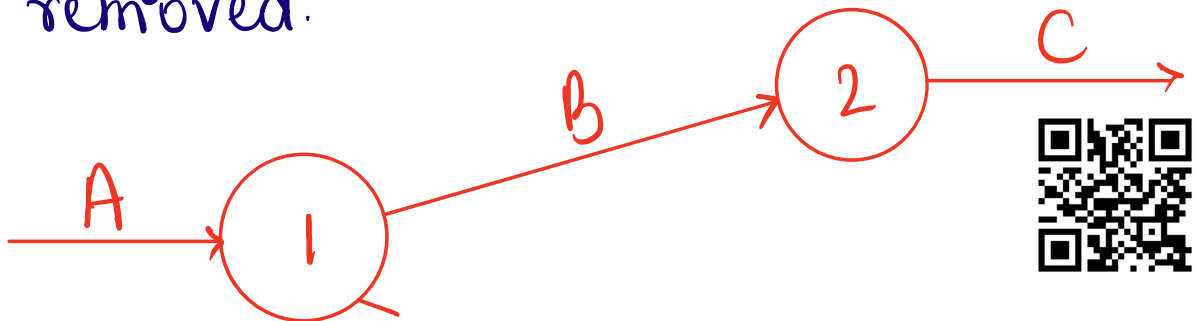


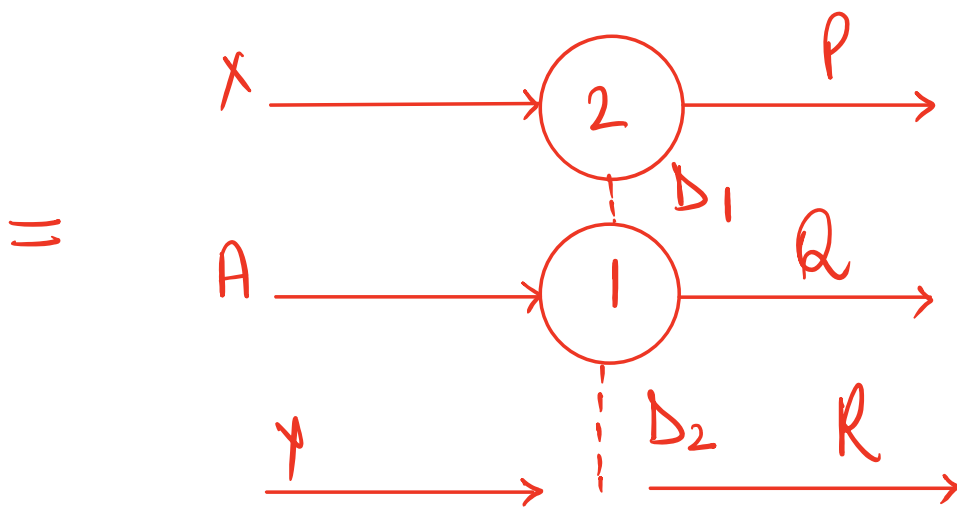
(ii)



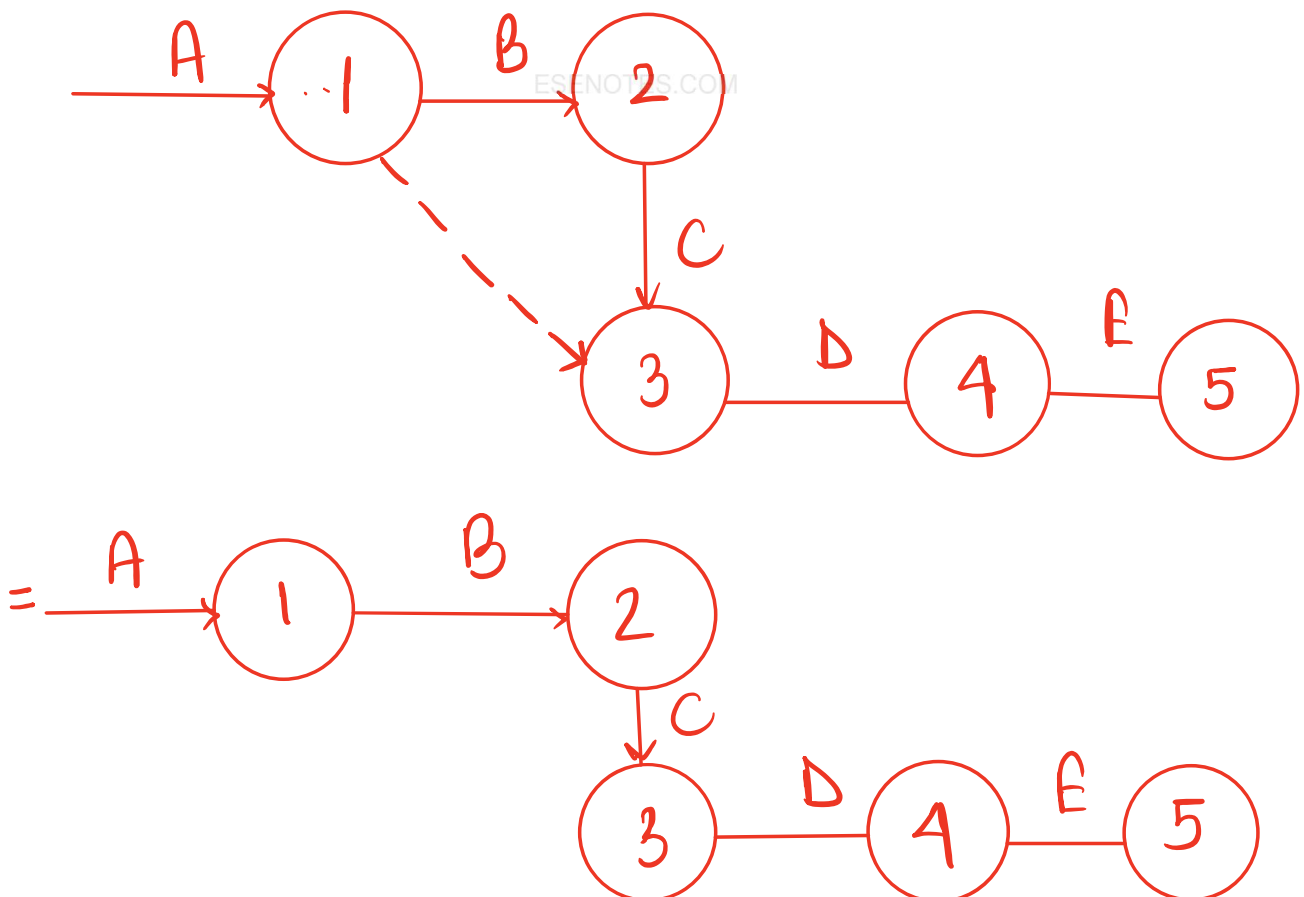


2. If dummy is the only activity terminating at its head event it can be removed.

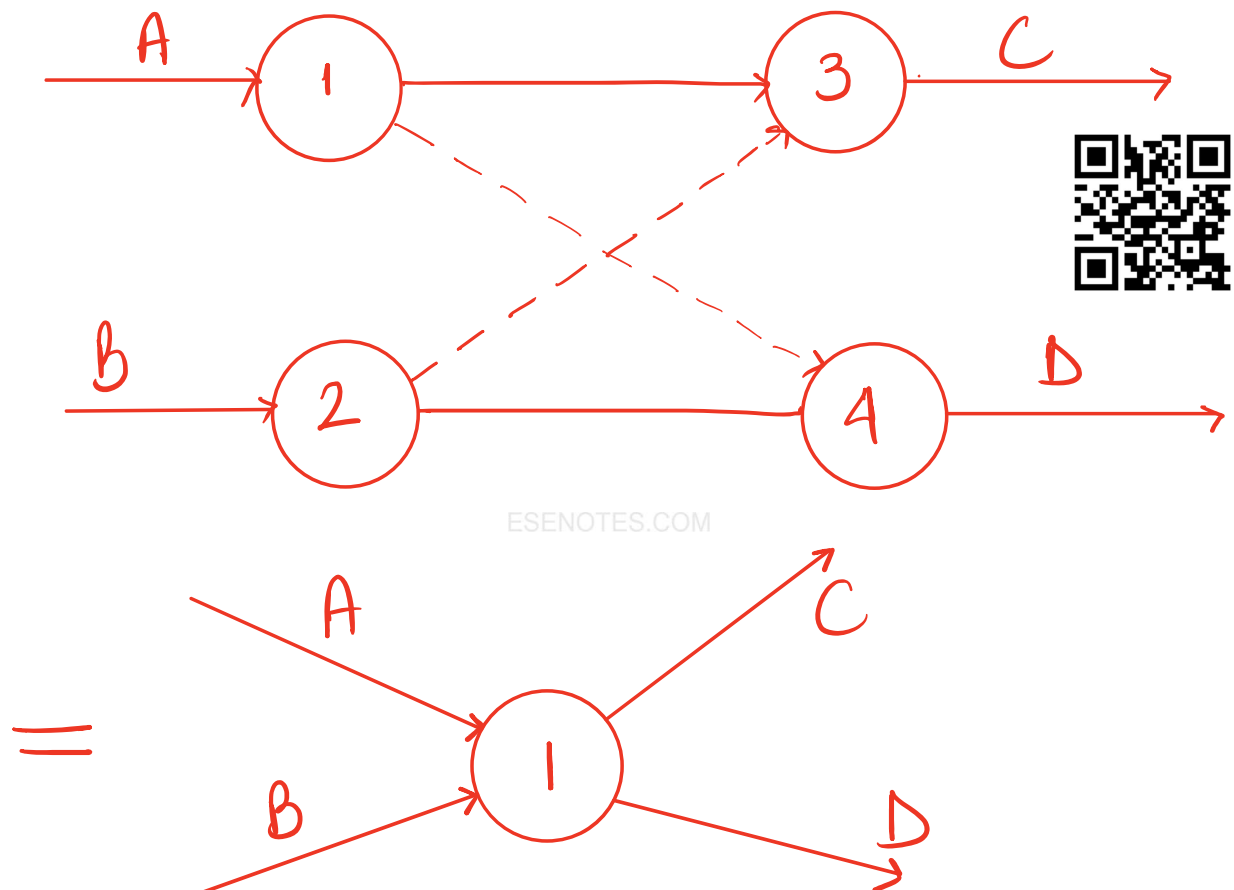




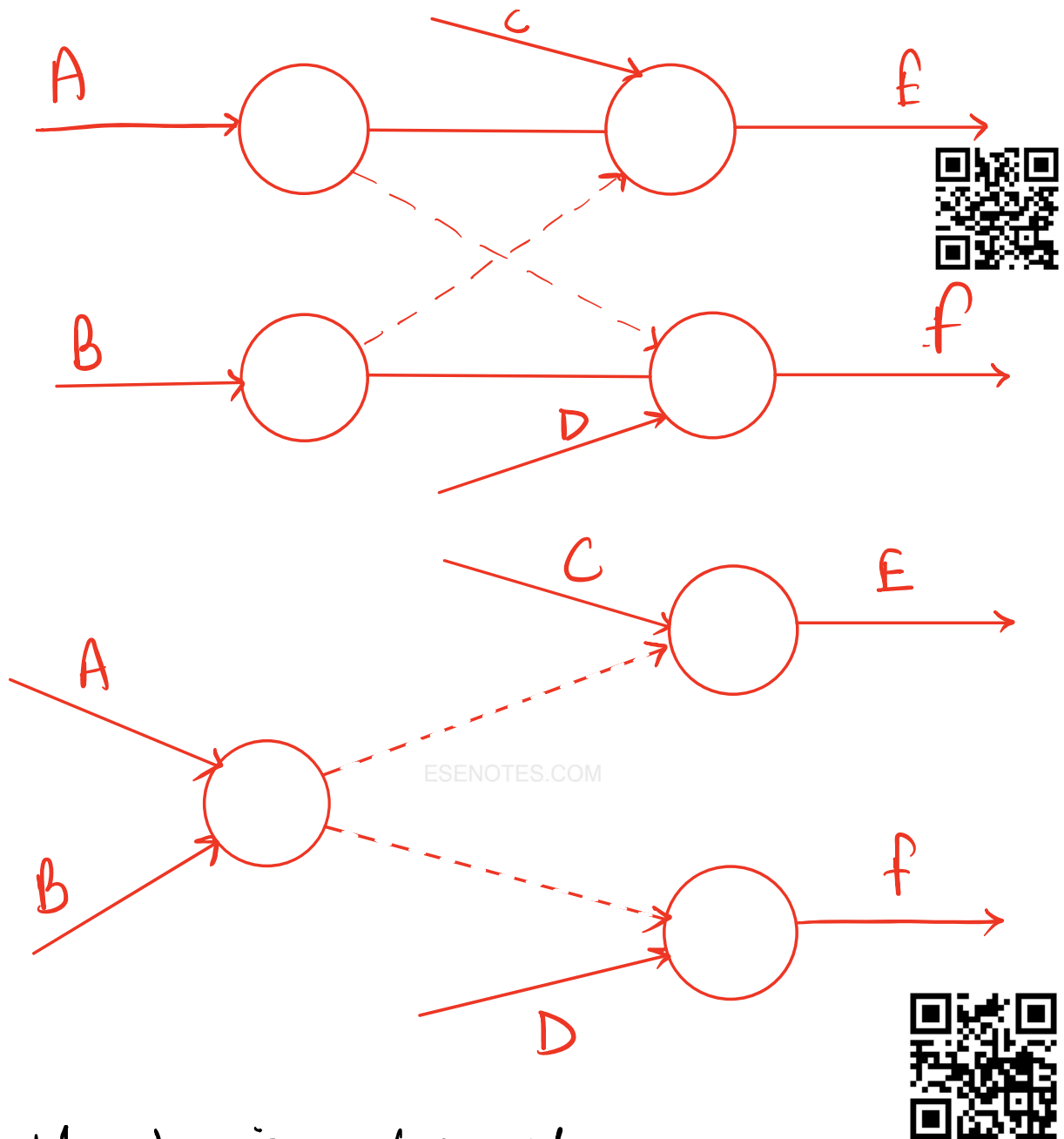
3. If dummy activity establishes the relationship which already exists in network, it can be removed.



4. If two or more activities emanate (start) from different nodes (events) having some set of predecessors activities, then the concerned activities start from same node.



5. If two or more activities have same set of successors activities which are also having other predecessors then the concerned activity must terminate at same node.



Numbering of event

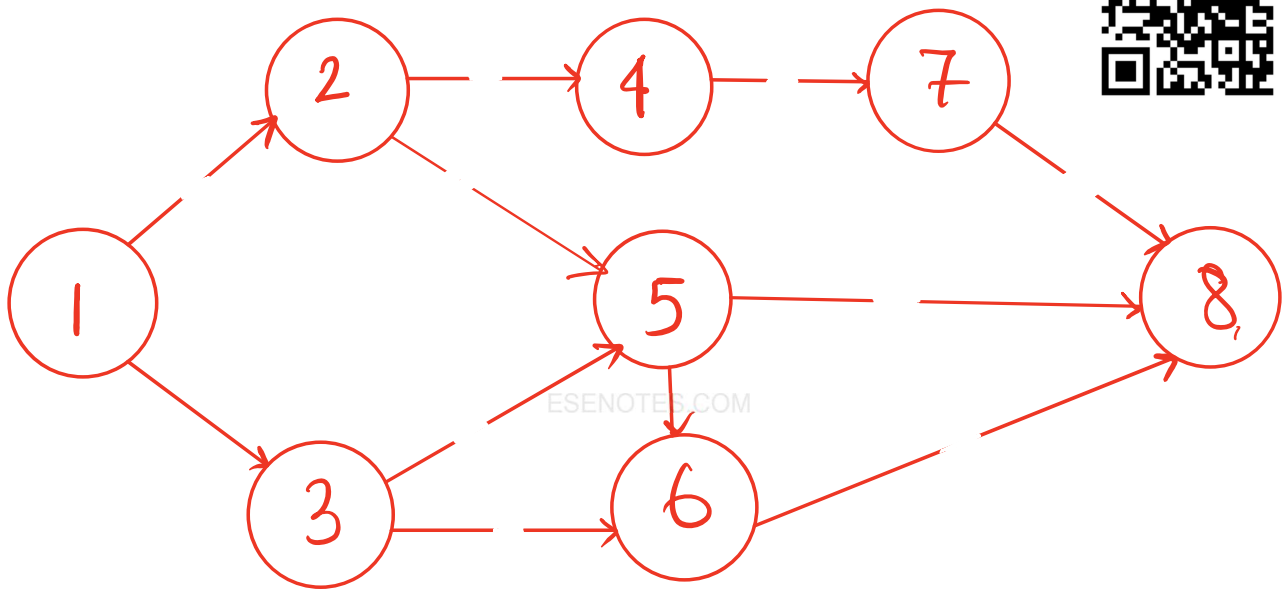
Numbering of event is done by Fulkerson's rule as follows.

1. No. the initial event as one ① (initial event has only outgoing arrows).
2. Remove all the outgoing arrows of

initial event. This will lead to formation of new initial event.

3. Number these new initial event sequentially.

4. Repeat step ② and ③ until all the events are numbered.



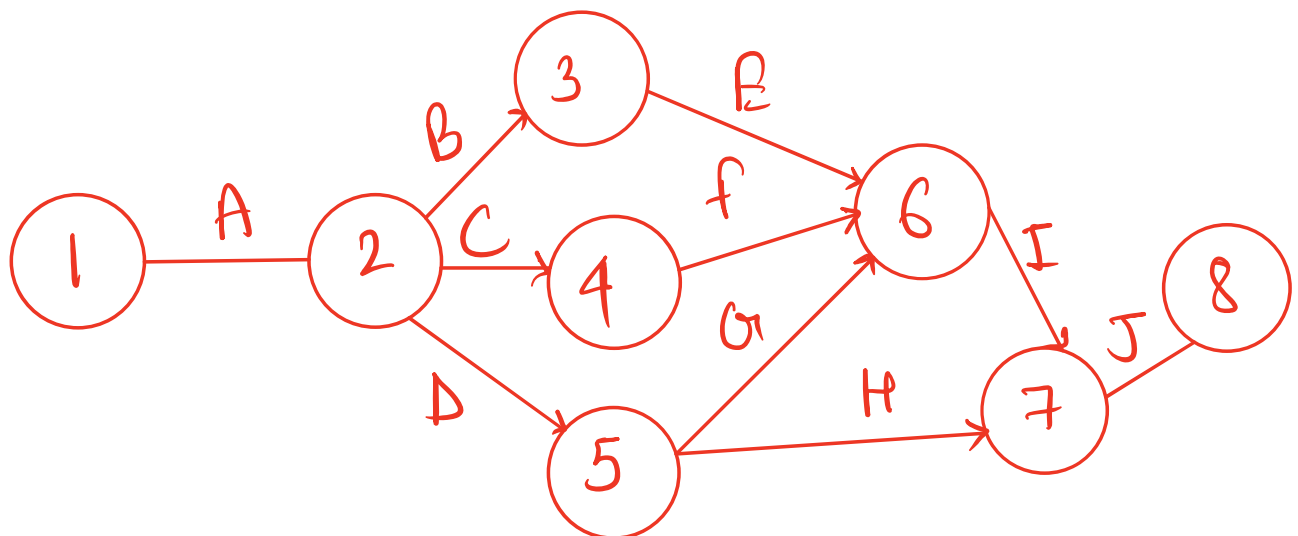
NOTE :

Skip numbering : in small network no modifications are required numbering is done in sequence. But in big network there frequent change, if numbering is done in sequence, huge time will be required for removing of events in case of modification.

In such case, skip numbering is adopted.

Example: A project consist of 10 activities as follows. Draw AOA network diagram and no. of events as the rule.

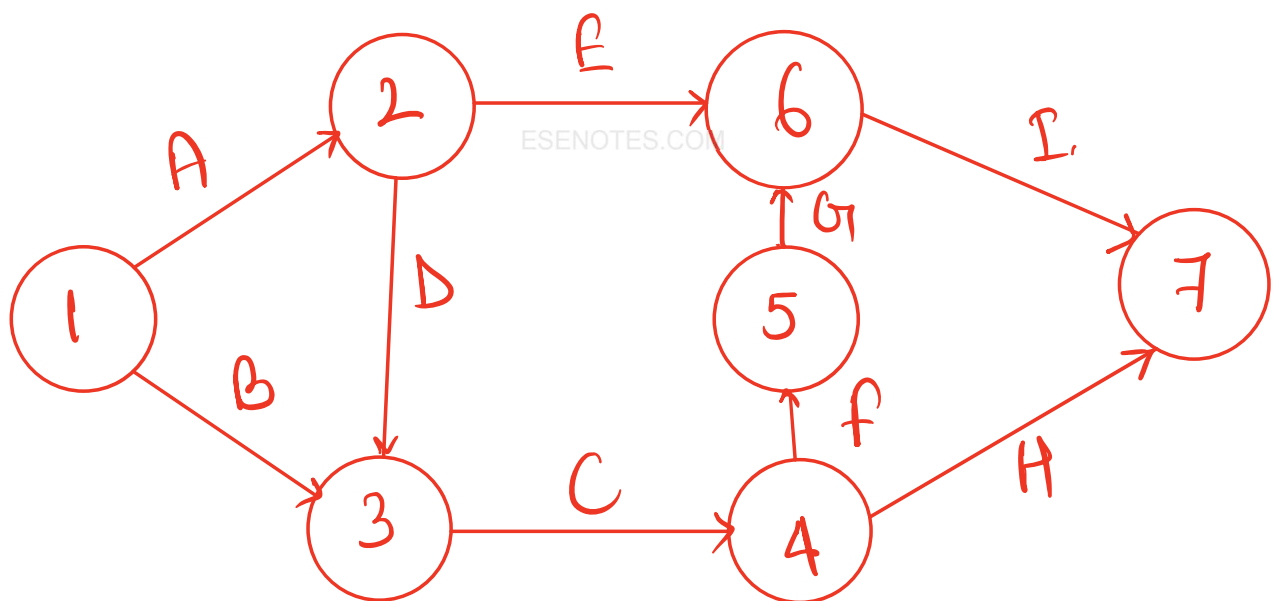
Job	Immediate
A	—
B	A
C	A
D	A
E	B
f	C
G	D
H	D
I	E, f, G
J	H, I



Question: Draw a network diagram for the project having following activity relationship



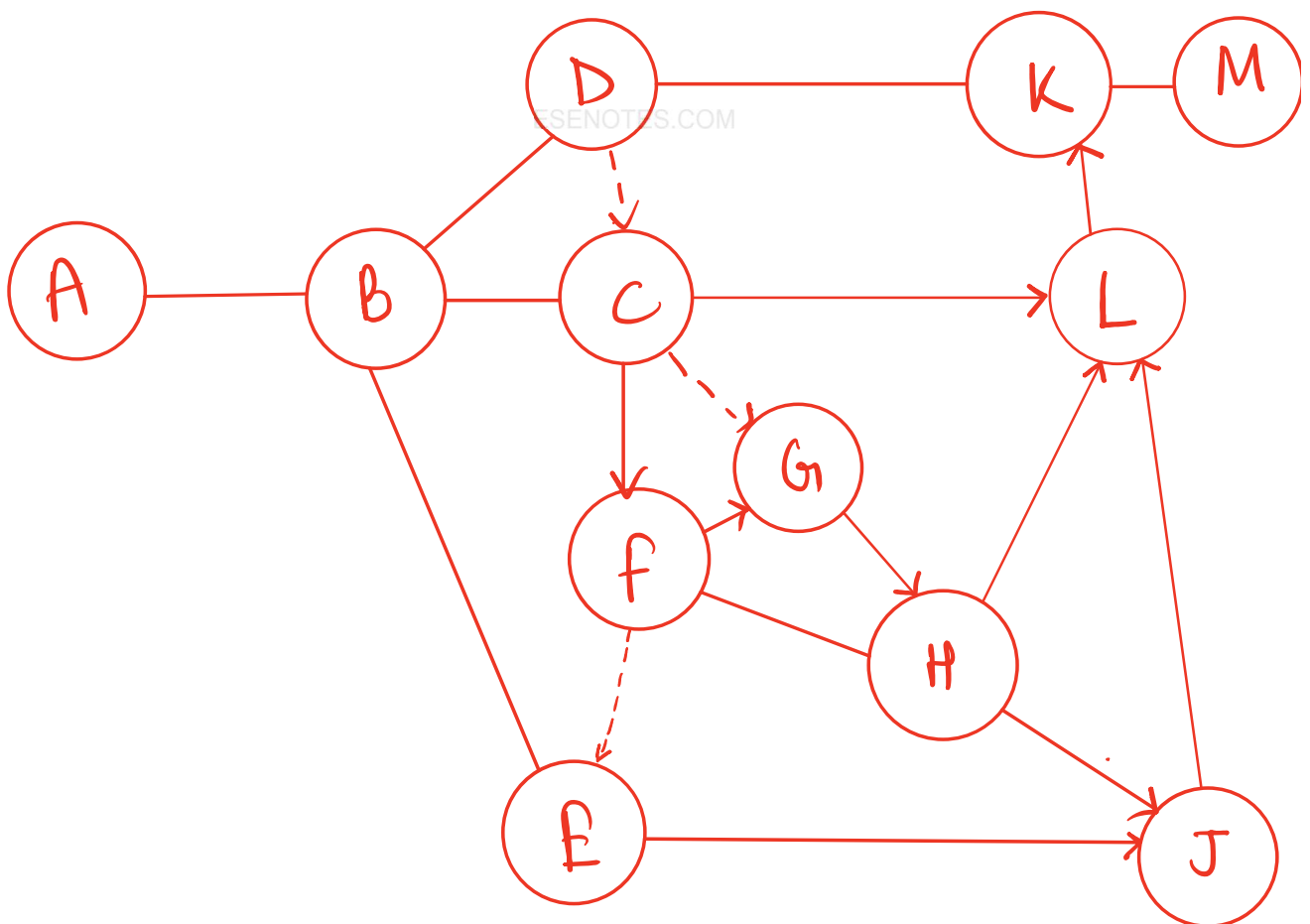
1. C follows B but preceeds F
2. C follows B but preceeds H
3. G follows f but preceeds I.
4. f follows A but preceeds I.
5. D follows A.
6. H and T terminates at same time
7. A and B start at same time.



Question: In a construction project, events have identified as A, B, C, D, E, f, G, H, J, K, L, M.

1. A is start event.

2. B occurs after A.
3. C succeeds B and precedes L but restrains the occurrence of G.
4. D occurs after B, before K and restrains E
5. f succeeds C, precedes H and restrain E.
6. E succeeds B, but precedes J.
7. H precedes L and restrains J.
8. L occurs after J, but before K.
9. M succeeds K, draw A pert network.
10. G succeeds f and precedes J.



Question : The given network shows the relationship between the activities P and w where P is plastering and w is white washing done by one set of labours.



If the entire work is to be executed in 3 - different section. Draw the network diagram for the following relationship.

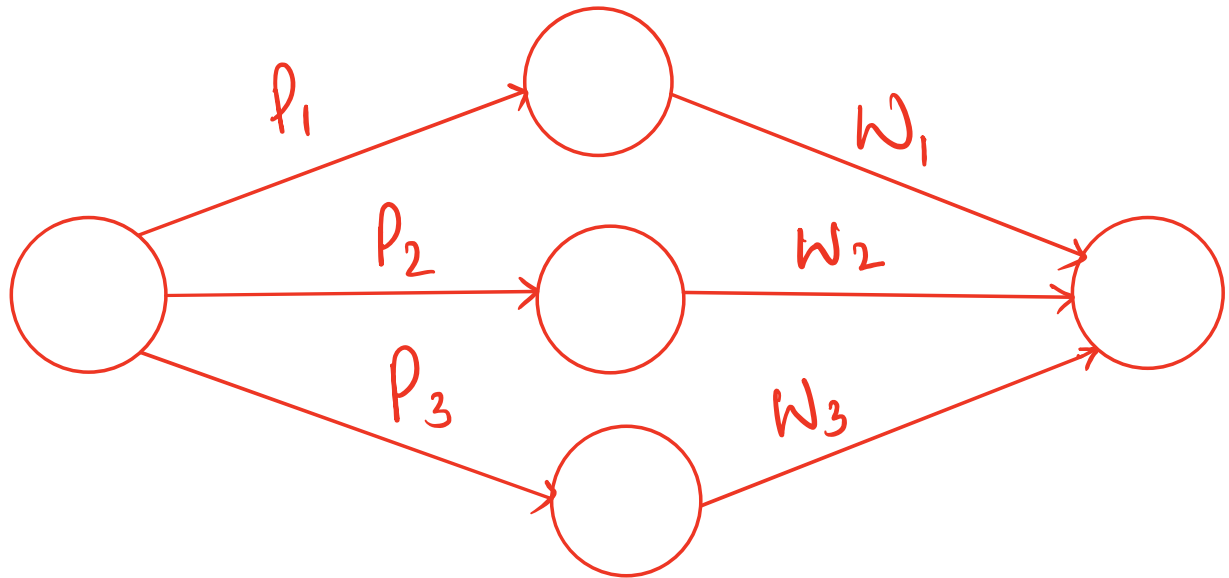
1. When P and w is available for all section.
2. P is available for 1-section and w is available for all 3 section.
3. P is available for 3 section and w is available for one section only.
4. When P and w are available for one section only.



Solution :

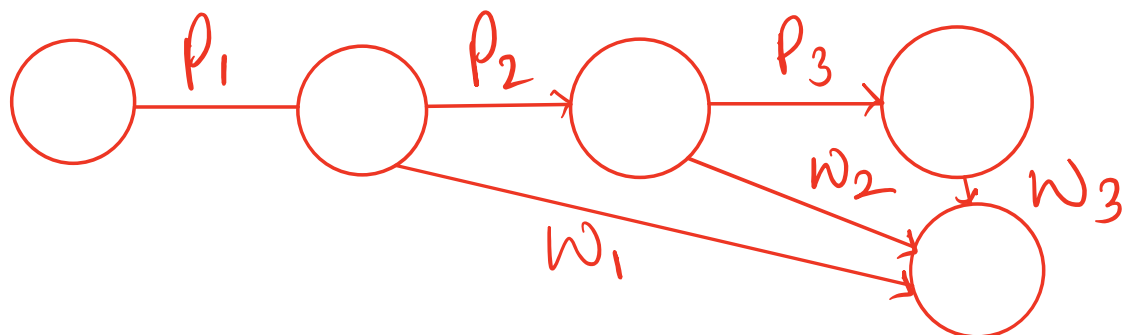
Activity	Pre. Activity.
P ₁	—
P ₂	—
P ₃	—
w ₁	P ₁ *
w ₂	P ₂ *
w ₃	P ₃ *

P_1	P_2	P_3
W_1	W_2	W_3



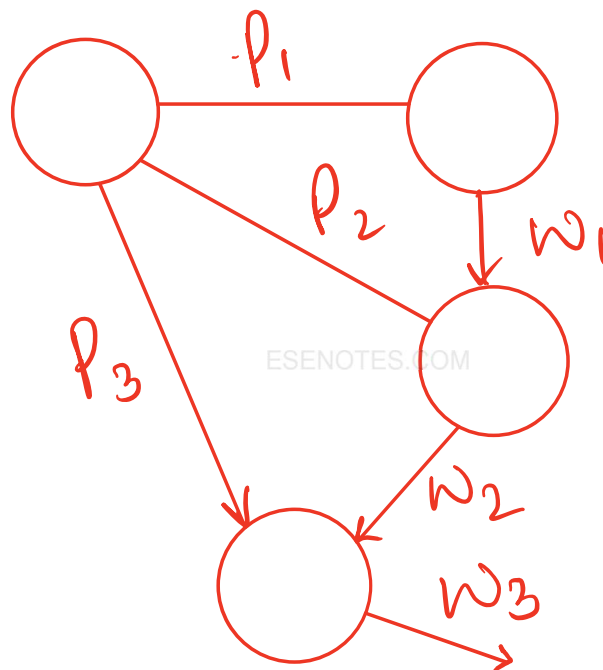
②

Activity	Pre. activity
P_1	—
P_2	P_1^*
P_3	P_2^*, P_1
W_1	P_1^*
W_2	P_2^*, P_1
W_3	P_3^*, P_2, P_1



III

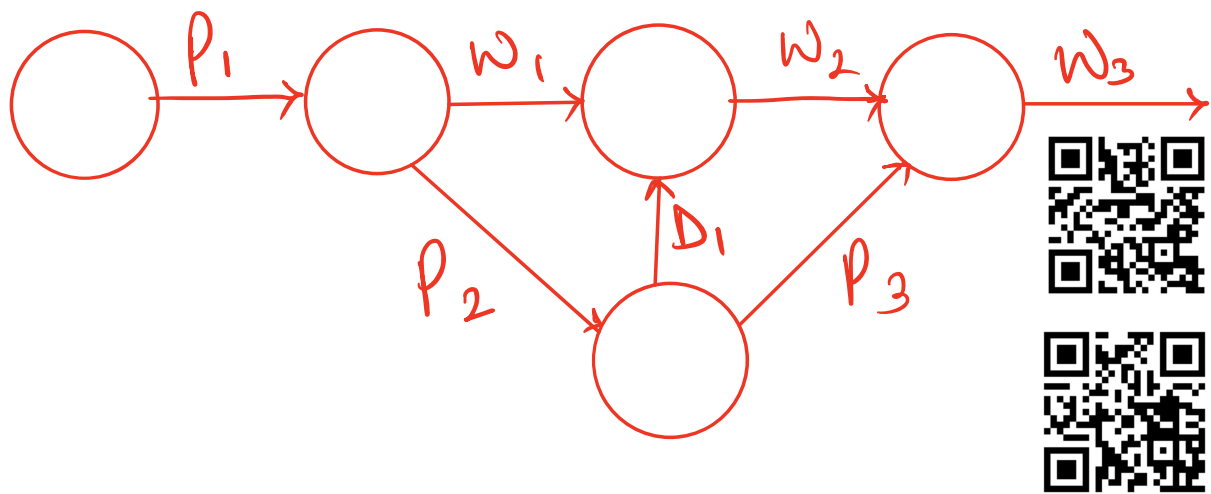
Activity	Pre. activity.
p_1	—
p_2	—
p_3	—
w_1	p_1^*
w_2	$p_2^* \quad w_1^* \quad p_1$
w_3	$p_3^* \quad w_2^* \quad p_2, w_1, p_1$



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IV

Activity	Pre activity.
p_1	—
p_2	p_1^*
p_3	$p_2^* \quad p_1$
w_1	p_1^*
w_2	$w_1^* \quad p_2^* \quad p_1$
w_3	$p_3^* \quad w_2^* \quad p_2 \quad p_1, w_1$



Question: A project consists of 3 operations, E, C and B where E stand for excavation for one set of group, C stand for concreting by other set of group and B stand for backfilling for another set of group. Each of 3 operation can be completed in 3 sections. Draw a network assuming that completion of 1 section in an operation allow the beginning of next operation.

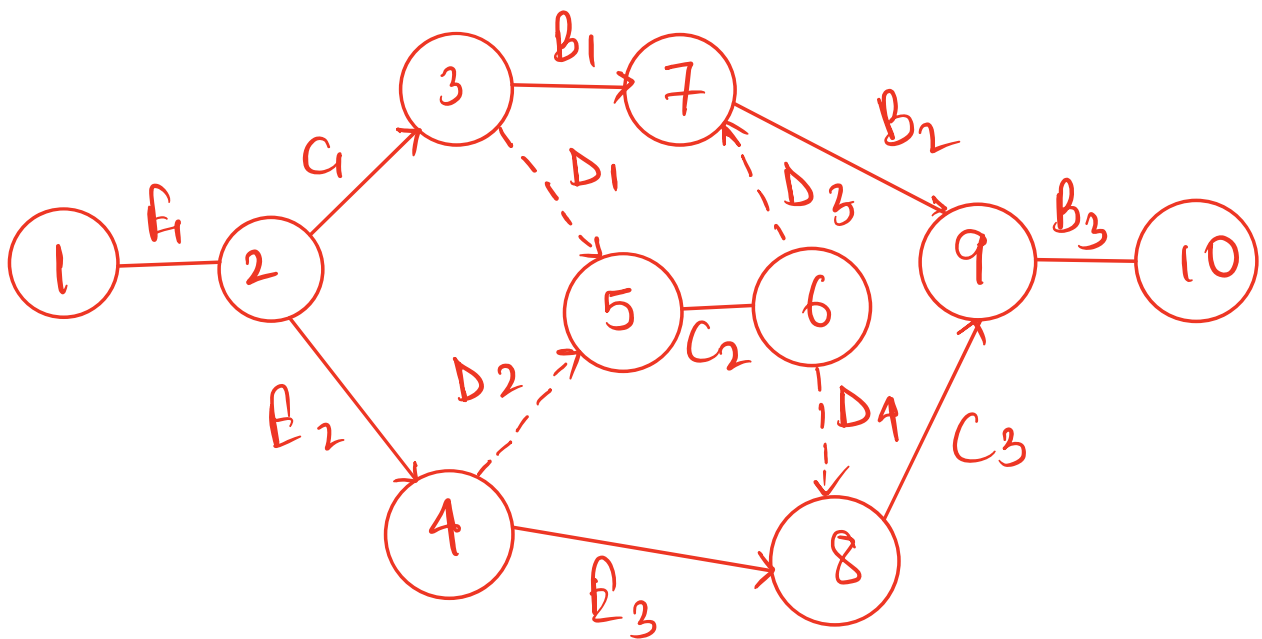
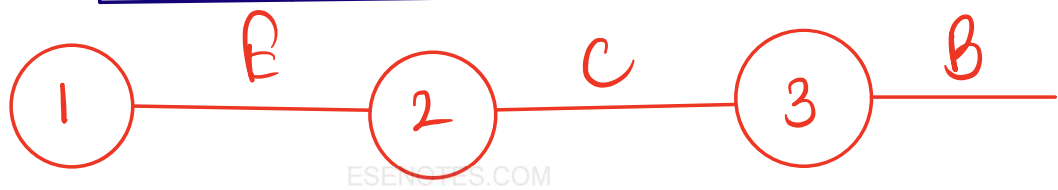
Solution:

Activity	Pre activity.
E ₁	—
E ₂	E ₁ *
E ₃	E ₂ *, E ₁
C ₁	E ₁ *

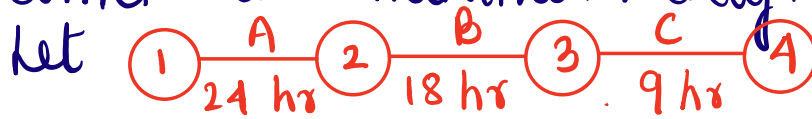


C_2	E_2^*, C_1^*, E_1
C_3	$E_3^*, C_2^*, E_2, E_1, C_1$
B_1	C_1^*, E_1
B_2	$C_2^*, B_1^*, E_2, C_1, E_1$
B_3	$C_3^*, B_2^*, E_3, C_2, E_2, E_1, C_1$

E_1	E_2	E_3
C_1	C_2	C_3
B_1	B_2	B_3

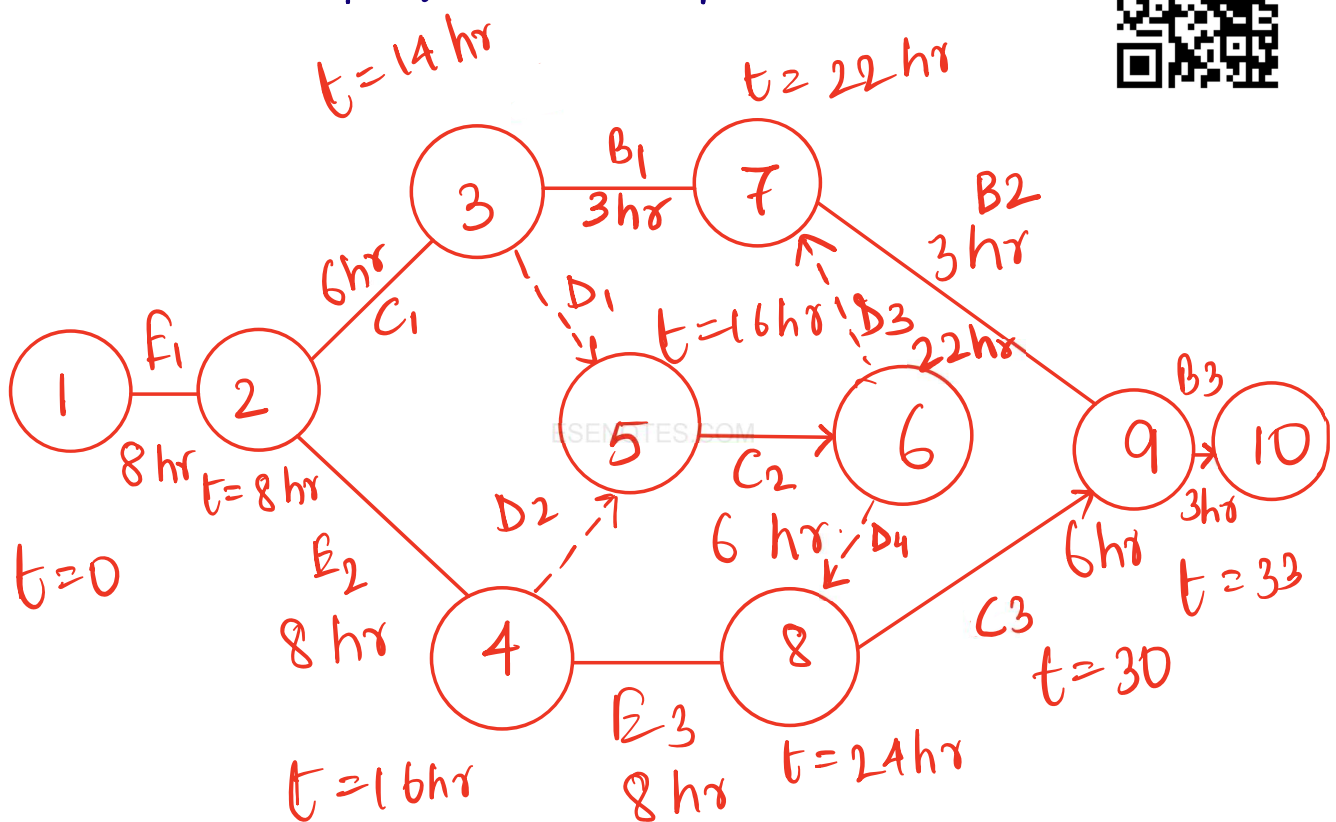


This network is laddered network. So, in this particular network completion of project comes out in minimum day.



$$T = 24 + 18 + 9 = 51 \text{ hr}$$

Divides the projects in 3 parts.



PREVIOUS YEAR QUESTION

ESE

Question 1: Which of the following techniques belong to 'Project Time Plan'?

1. Critical path method
 2. Precedence network analysis
 3. Line of balance technique
 4. Linear programme chart
- a) 1, 2 and 3 only b) 1, 2 and 4 only
c) 3 and 4 only d) 1, 2, 3 and 4.
- (2019)

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Solution: (d)

Techniques for project time plan are critical path method, precedence network analysis and line of balance technique.

Linear programme chart is used for profit maximisation, cost minimisation and resources allocation.

Question 2: The graphical representations wherein long duration jobs are

broken down to key segmental elements, wherein events are shown in chronological order without attention to logical sequencing and wherein interdependencies between the events is not highlighted, is referred to as

- a) CPM
- b) Milestone chart
- c) GANTT chart
- d) PERT

(2019)

Solution : (b)

Question 3: Which one of the following techniques is not covered in Project Network Analysis?

- a) Critical Path Method
- b) Program Evaluation and Review Technique
- c) Procedure Network Analysis
- d) Measurement book

(2020)

Solution : (d)

PERT (Programme Evaluation and Technique)



It is a technique which is used for planning scheduling and controlling the project.

- The most important factor in completion of project is time involved in it, which depends upon the estimate of different activities.
- Time estimate for the activity can be done by two methods.
 1. Deterministic approach : Sufficient knowledge about completion of activity.
 2. Probabilistic approach : Here not much idea about completion time of an activity is known.
- PERT follows probabilistic approach and it is an event oriented technique.
- In order to account for uncertainty involved in completion of activity 3-time estimates are considerable.

1. Optimistic time estimate : Min. time required for completion of activity in most ideal condition.

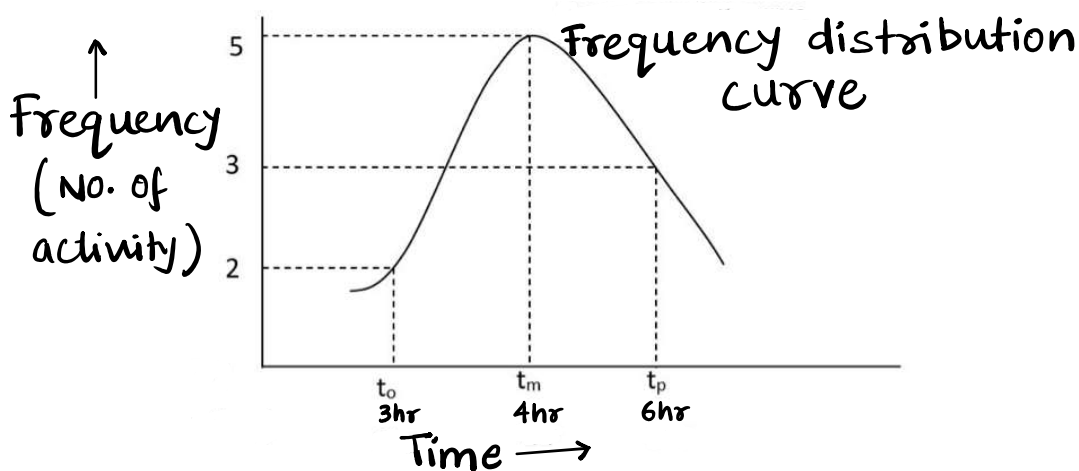


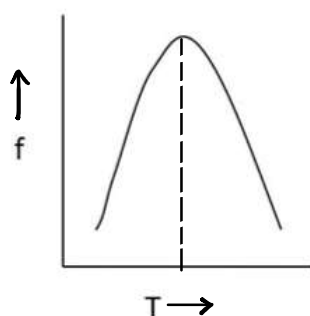
2. Pessimistic time estimate : Maximum time required for completion of activity in worst condition while analysing worst condition, contingencies are not considered.

3. Most likely time : It is the time required for completion of activity in normal condition.

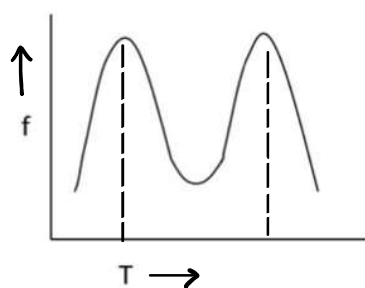
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- Time estimate can be represented in the form of frequency distribution curve.

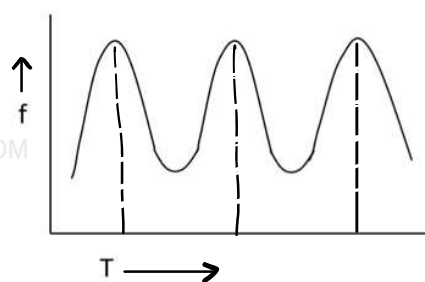




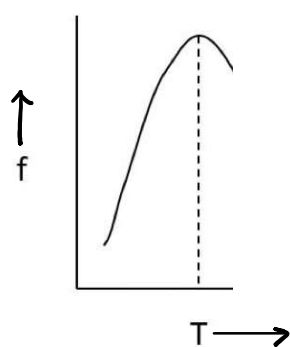
(i) Uni-model



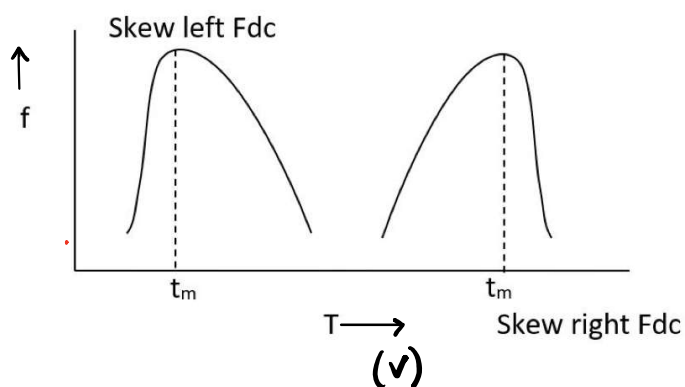
(ii) Bi-model Frequency distribution curve



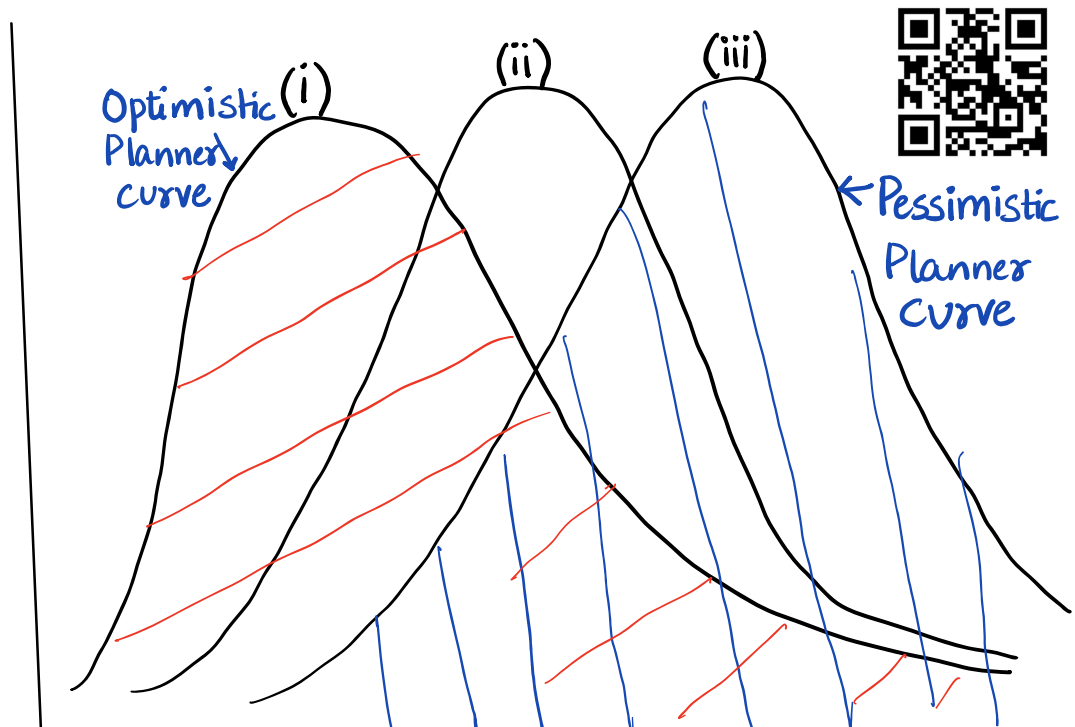
(iii) Multi model Frequency distribution curve



(iv) Symmetric



(v)



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NOTES:

Properties of frequency distribution curve:

1. Mean time $\bar{X} = \frac{X_1 + X_2 + \dots + X_n}{n}$
2. Deviation / Variation $D/V = |X_i - \bar{X}|$
3. Variance $= \sigma^2 = \sum \frac{D^2}{n} = \frac{\sum (X_i - \bar{X})^2}{n}$
4. Standard deviation $= \sigma = \sqrt{\frac{(X_i - \bar{X})^2}{n}}$

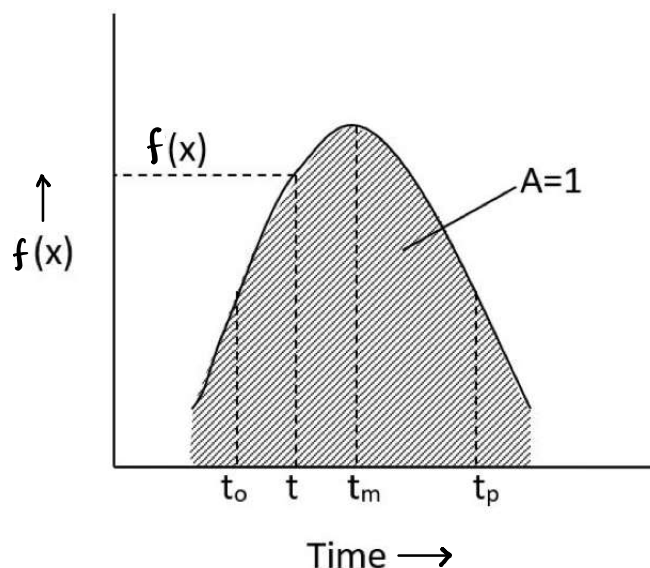
All these are parameters of spread or dispersion.



Probability Distribution:

In frequency distribution we analyse group behaviour but in probability distribution we analyse the distribution of probability values of diff / all possible outcomes.

Since probability values varies from 0 to 1 its analysis is comparatively simpler, hence it is preferred over frequency distribution.



A probability distribution curve is plot of function $f(x)$ termed as Probability distribution function w.r.t to time. Height of which is standardized such that area under this curve is unity.



$$\begin{aligned} A &= \int y dx \\ &= \int_{-\infty}^{\infty} f(x) dx \\ &= 1. \end{aligned}$$

NOTE:

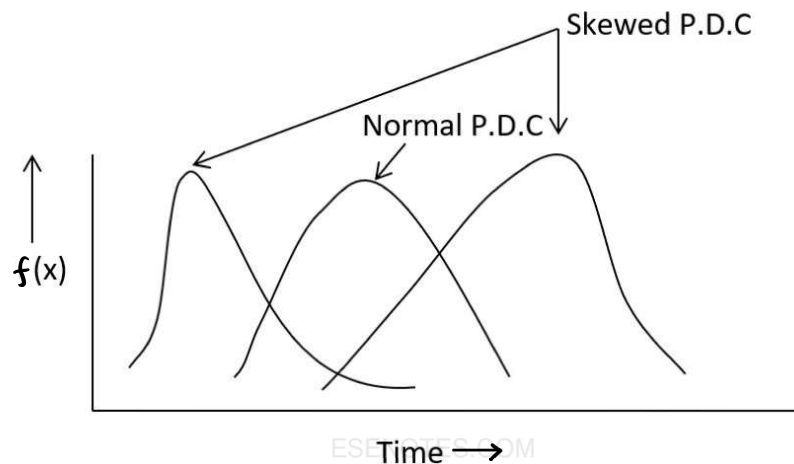
The ordinate of this curve for any abscissa (x) does not gives probability but function $f(x)$.

Hence, probability at any point x is given by ratio of area of curve upto that point and total area of the curve.

$$P(x) = \frac{\text{Area of curve upto } x}{\text{Total area.}}$$

$$= \frac{\int_{-\infty}^x f(x) dx}{\int_{-\infty}^{\infty} f(x) dx} = \int_{-\infty}^x f(x) dx.$$

if P.D.C is symmetric about apex it is termed as normal / Gaussain/Bell shaped curve and it is not symmetric about apex it is termed as β -distribution curve.

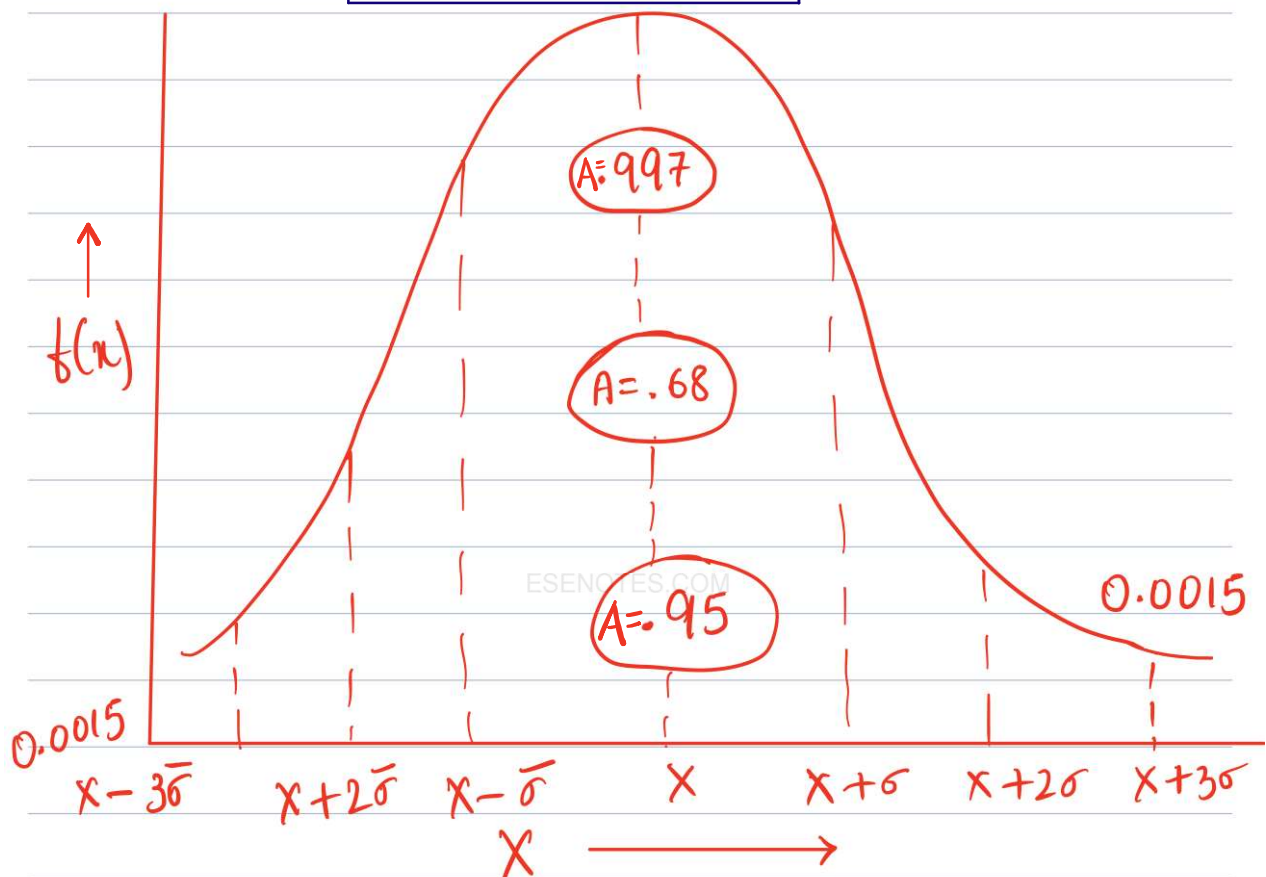


following properties of normal probability distribution curve are significant.

Limits	Area %	
$-\infty \rightarrow \bar{X} - 3\sigma$	0.15	$\left. \begin{aligned} f_o &= \bar{X} - 3\sigma \\ f_p &= \bar{X} + 3\sigma \end{aligned} \right\} \begin{array}{l} \text{Assumptions} \\ \text{error at 3\%} \end{array}$
$-\infty \rightarrow \bar{X}$	50	
$\bar{X} \pm \sigma$	68	
$\bar{X} \pm 2\sigma$	95	
$\bar{X} \pm 3\sigma$	99.7	$t_p - t_o = (\bar{X} + 3\sigma) - (\bar{X} - 3\sigma)$ $t_p - t_o = 6\sigma$

$$\Rightarrow \boxed{\sigma = \frac{t_p - t_o}{6}}$$

$$\boxed{\sigma^2 = \left(\frac{t_p - t_o}{6} \right)^2}$$



In PERT each activity is assumed to follow β -distribution and entire project is assumed to follow normal probability distribution curve.

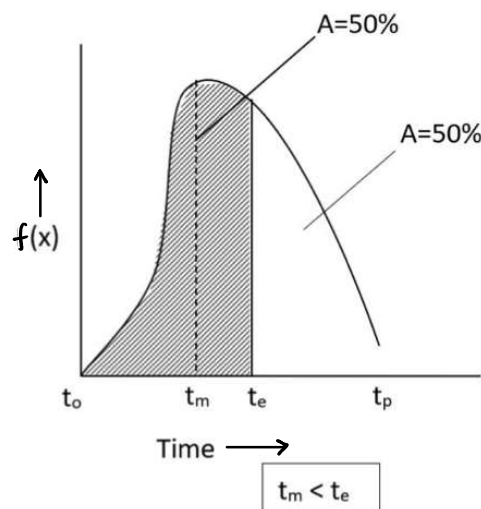
— Expected time of activity it is an over-

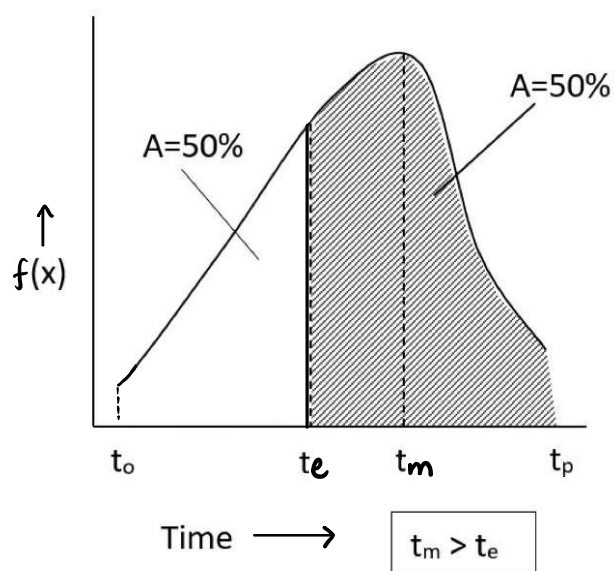
age or mean time in which an activity can be completed. It is the time in which 50% chance exist for completion of an activity.



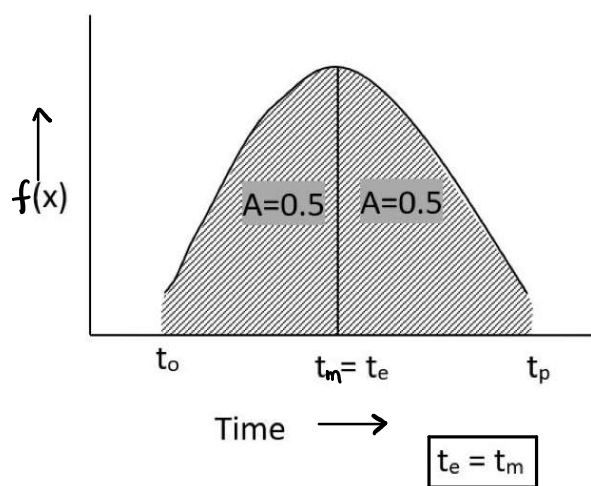
- If the shape of probability distribution curve is known for activity is expected time can be calculated directly. (But practically it is not known). Hence, this time is computed by taking weighted by

$$t_e = \frac{t_o + t_p + 4t_m}{6}$$

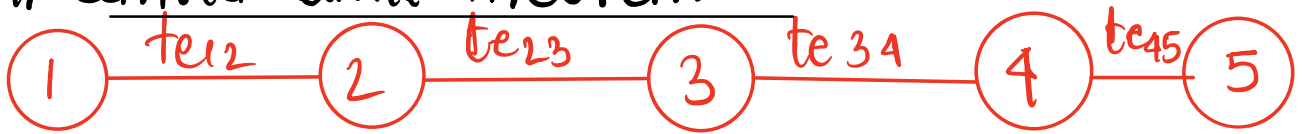




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Central limit theorem



In a network, there are no. of activities in series, then the expected time of the project along particular path is summation of expected time of all the activity along the path.

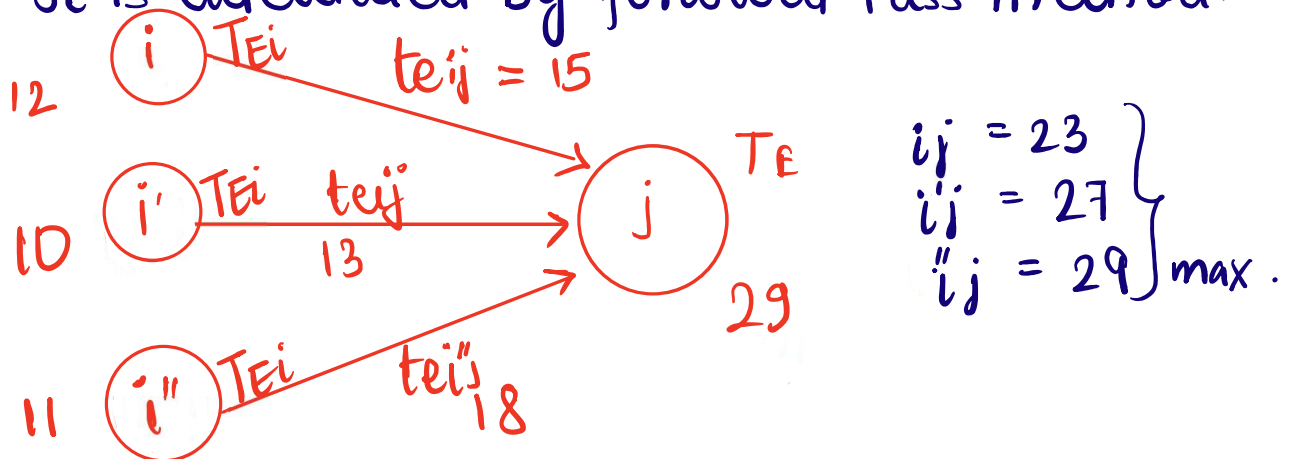
$$(T_e)_{\text{project}} = \sum t_{eij} \text{ along the particular path.}$$



Event time

1. Earliest Expected Event time (T_E)

- It is the minimum time in which an event can occur.
- An event occurs when all the activities leading to it are completed.
- It is calculated by forward Pass method.



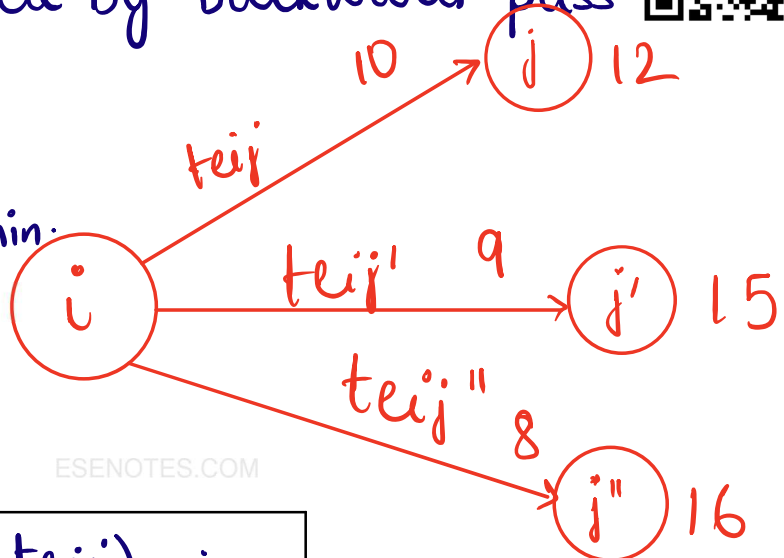
$$T_{Ej} = \{(T_{Ei}) + (t_{Eij})\}_{\max}$$

2. Latest Allowable Occurrence time (T_L)

It is max. time in which an event may occur without delaying project.

— It is calculated by backward pass method.

$$\left. \begin{aligned} i_j &= 12 - 10 = 2 \\ i_{j'} &= 15 - 9 = 6 \\ i_{j''} &= 16 - 8 = 8 \end{aligned} \right\} \min.$$



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$$T_{Li} = T_{Lj} - (t_{Eij})_{\min}$$

Slack

It is the time by which occurrence of an event can be delayed without affecting the schedule completion time of project.

It is mathematical diff between latest allowable occurrence and earliest expected event occurrence time.

$$S = (T_L - T_E).$$

The value of the slack can be +ve, 0, or

-ve.

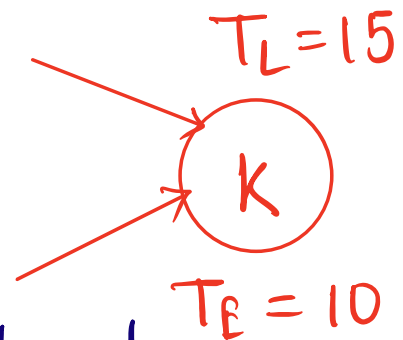
A. If $\text{Slack} > 0$

$$S = T_L - T_E > 0$$

$$T_L > T_E$$

+ve slack indicates

1. The events are running ahead of the schedule hence can be delayed by time value equal to slack without delaying project.
2. These events are termed as sub-critical event.



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B. If $\text{Slack} = 0$

$$S = T_L - T_E = 0, T_L = T_E$$

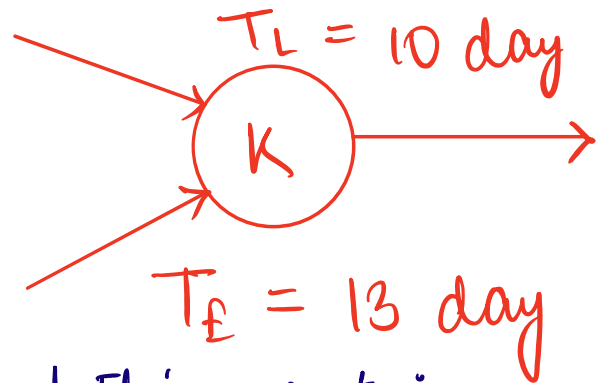
1. Slack indicates that this event is on schedule and we can not afford the delay in the occurrence of this event as delay in occurrence of this event would delay the entire project.
2. These events are termed as critical event.

C. If $\text{Slack} < 0$

$$S = T_L - T_E < 0$$



$$T_L < T_E$$



- (-)ve slack indicates that this event is running behind the schedule.
- These events are termed as supercritical event.



Critical Path

Critical Path is the longest path in terms of time in network in which the project can be completed.

- Critical path is shortest path in terms of time in which project can be completed as early as possible.
- Critical path is that path which joins critical event. (C.E for which slack is zero).

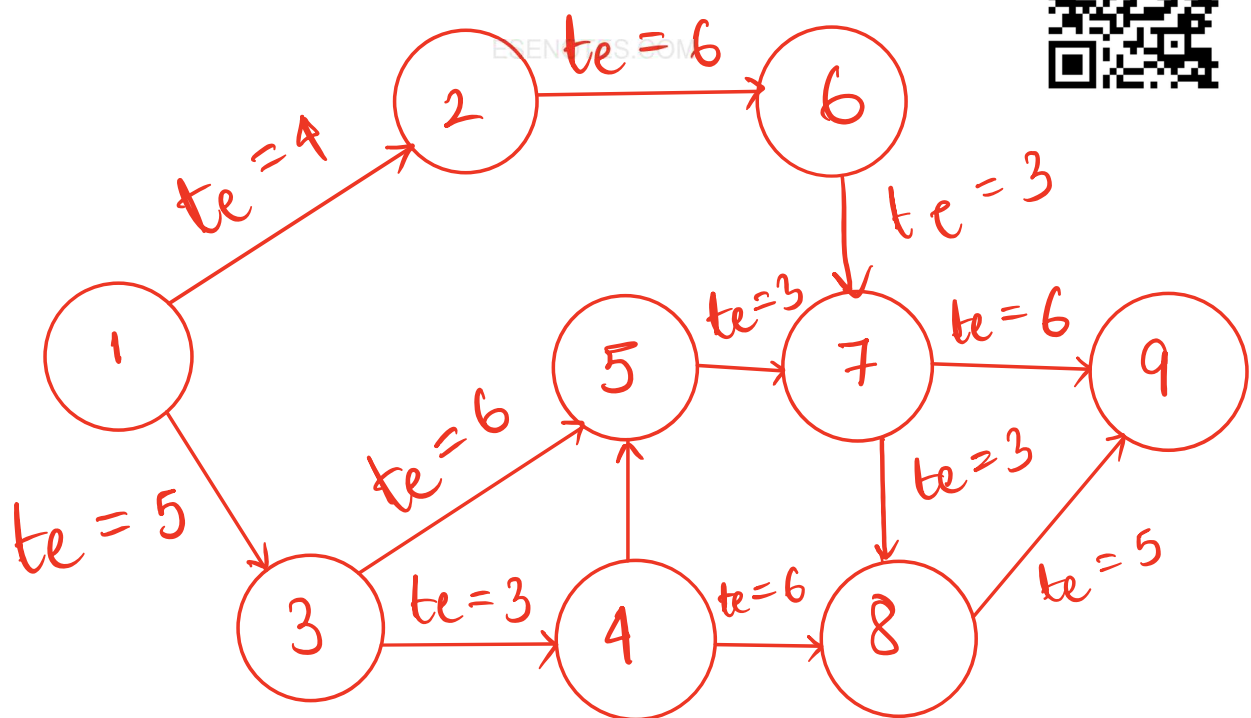
NOTE:

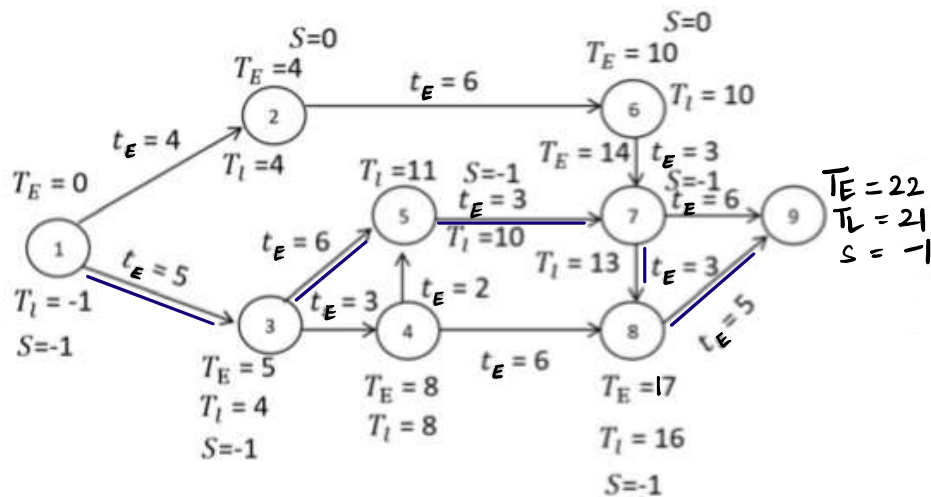
- In case a network has multiple critical path. Then emphasis is made on that critical path whose variance or standard deviation is more.

NOTE :

As per central limit theorem variance of any path/critical path it will be equal to sum of variance of activities along that path or critical path.

Question : The expected time of completion in days for each activities of network is shown in the fig. Determine the critical path if scheduled completion time is 21 days.





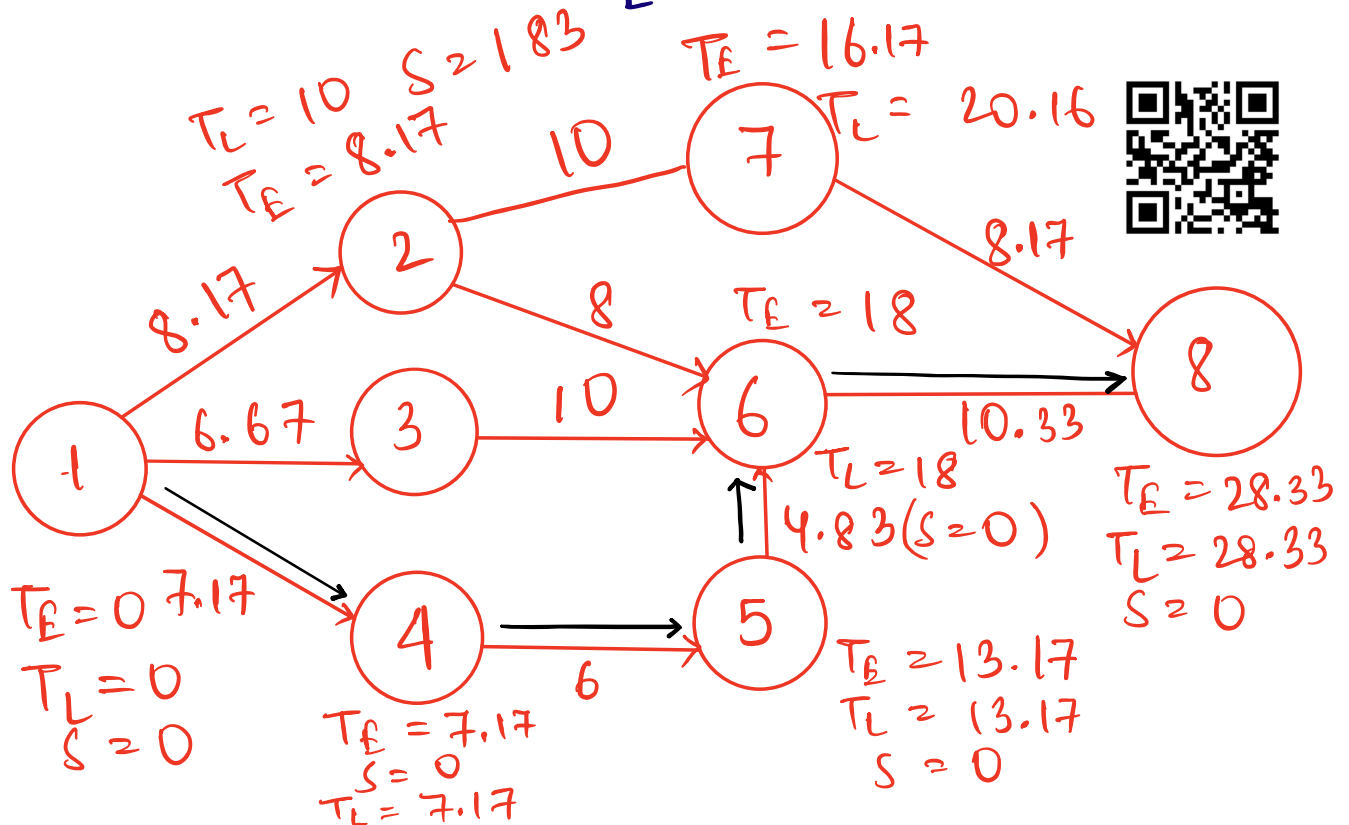
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Critical path, 1-3-5-7-8-9 $\Rightarrow 22$ days = T_E .

Question: The network for a certain project is given, find its critical path.

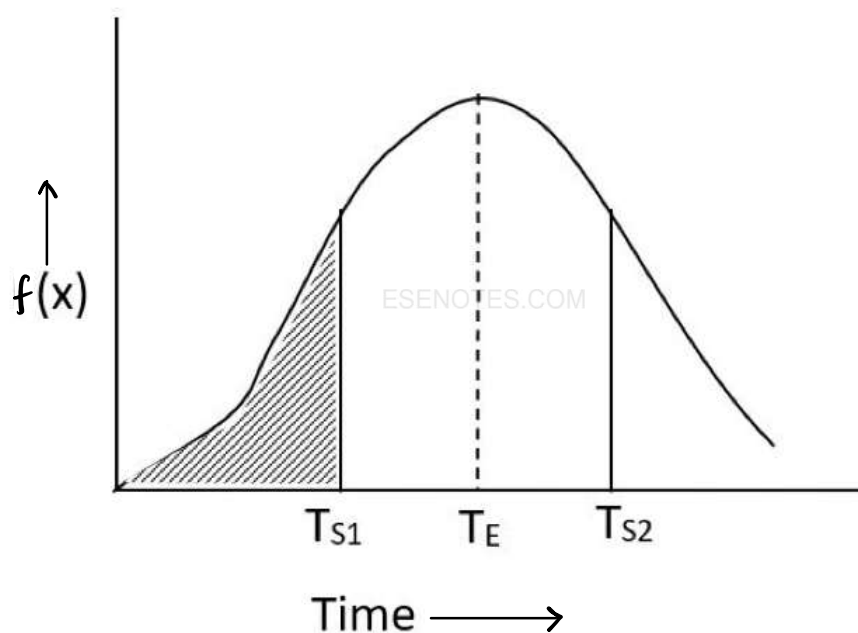
Activity	t_o	t_m	t_p	t_L
1-2	6	8	11	8.167
1-3	3	7	9	6.67
1-4	5	7	10	7.17
2-7	8	10	12	10
2-6	4	8	12	8
3-6	8	10	12	10
4-5	4	6	8	6
5-6	3	3	6	4.63
7-8	5	8	12	8.17
6-8	7	10	15	10.33

Critical path = 1-4-5-6-8
 $T_E = 28.33$





Probability of completion of project



$$\begin{aligned} P(T_{S1}) &= \text{Area}(T_S) = \text{Area } T_{E2} - \text{Area}(T_E - T_{S1}) \\ &= 0.5 - \text{Area}(T_E - T_{S1}) \\ P(T_{S1}) &\propto \text{Area}(T_E - T_{S1}) \end{aligned}$$

$$\begin{aligned}
 P(T_{s2}) &= \text{Area}(T_{s2}) \\
 &= \text{Area}(T_E) + \text{Area}(T_{s2} - T_E) \\
 &= .5 - \text{Area}(T_E - T_{s2}) \\
 P(T_{s2}) &\propto \text{Area}(T_E - T_{s2}).
 \end{aligned}$$



- The probability of completion of the project in schedule time is computed as follows.

STEP ①

Determine standard deviation of project along critical path (Use central limit Theorem).

$$\sigma^2 = \sigma_1^2 + \sigma_2^2 + \dots + \sigma_n^2$$

Subcritical activities.

$\sigma = \sqrt{\text{Sum of square of standard deviation of critical activity or variance.}}$

STEP ②

Determine $(T_s - T_E)$ and expressed it in terms of parameter i.e., probability factor or normal deviate (Z).

$$Z = \frac{T_s - T_E}{\sigma}$$

and this Z is linked with probability.

Z	0	1	2	3
Probability	50	84.1%	97.7%	99.9%

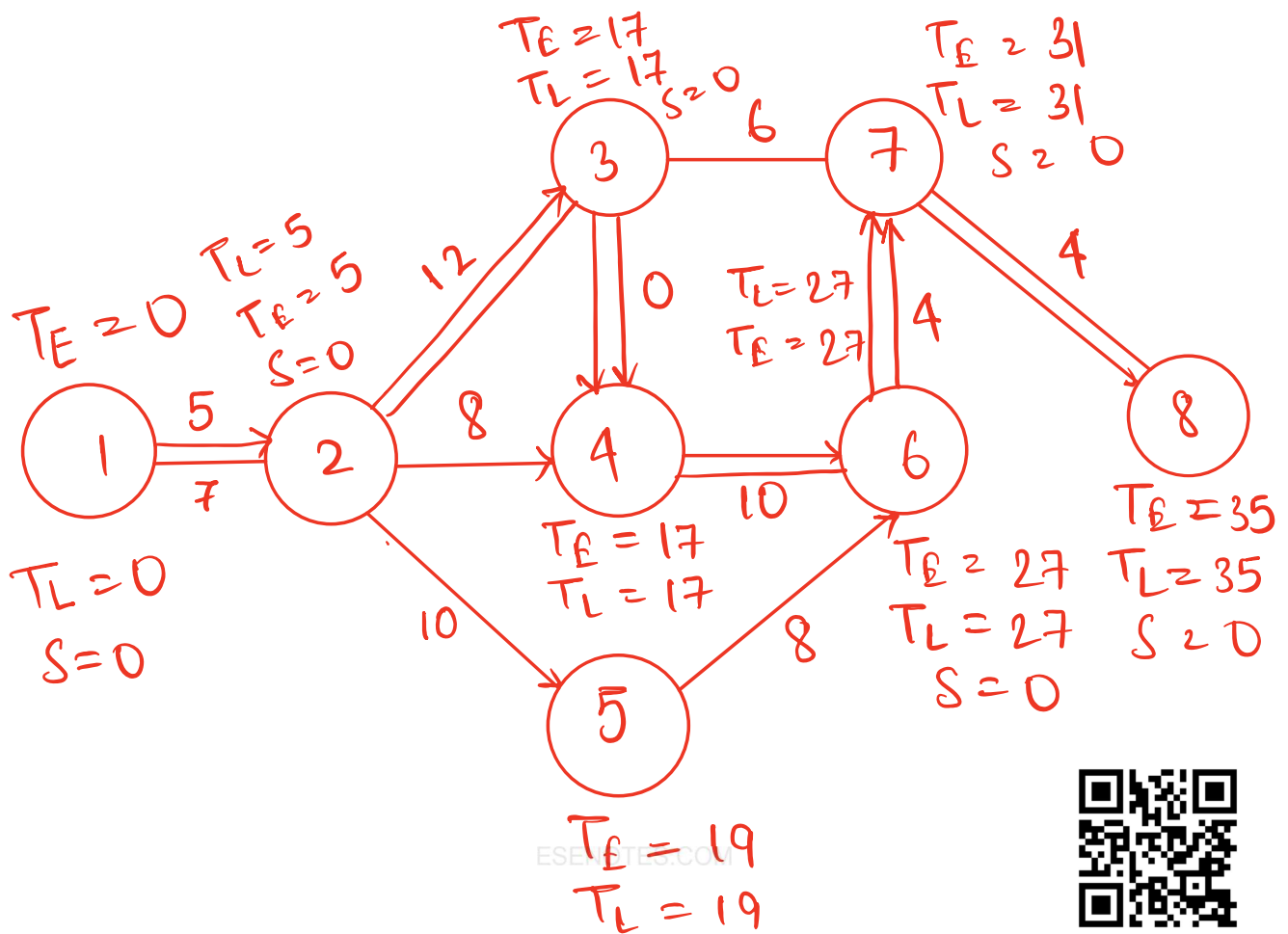
Z	-1	-2	-3
	15.9	2.3	0.1



Question : for given project determine critical path, standard deviation. Compute the time duration that will provide 90% probability of its completion in time.
- Also compute probability of completion of project in 30 days.

Activity	t_o	t_m	t_p	t_e	$\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$
1-2	2	5	8	5	1
2-3	8	11	20	12	4
2-4	4	7	16	8	
2-5	4	9	20	10	
3-4	0	0	0	0	0
3-7	3	5	13	6	
4-6	7	10	13	10	1
5-6	3	7	17	8	
6-7	2	3	10	4	1.77
7-8	2	4	6	4	0.44

$$T_L = 31$$



(i) Critical Path: 1-2-3-4-6-7-8

$$\sigma = \sqrt{\sigma_1^2 - 2 + \sigma_2^2 - 3 + \sigma_3^2 - 4 + \sigma_4^2 - 6 + \sigma_6^2 - 7 + \sigma_7^2 - 8}$$

$$(ii) \sigma = \sqrt{1 + 4 + 0 + 1 + 1.77 + 0.44}$$

$$\sigma = 2.86$$

(iii)

Z	P
1	84.1
2	97.7

→ 95% P

$$Z = 1.801$$

$$1.801 = \frac{T_s - 35}{2.86}$$

$$T_s = 40.15 = 41 \text{ days}$$

$$(iv) T_s = 30 \text{ days}, Z = \frac{T_s - T_E}{\sigma}$$

$$= \frac{30 - 35}{2.86}$$

$$= -1.75$$

Z	p
-1	15.9
2	2.3

$$p = 15.9 + \left[\frac{15.9 - 2.3}{-1 + 2} \right] \times (-1.75)$$

$$p = 5.7\%$$

PREVIOUS YEAR QUESTIONS

ESE

Question 1: It is estimated that an activity can be assigned an optimistic duration of 16 days, a pessimistic duration of 28 days and a most likely duration of 19 days. What is the expected duration for this activity?

- a) 20 days
- b) 22 days

- c) 19 days
- d) 18 days

(2015)

Solution: (a) 20 days

$$t_o = 16, t_m = 19, t_p = 28$$

$$t_e = \frac{t_o + 4t_m + t_p}{6}$$

$$t_e = \frac{16 + 4 \times 19 + 28}{6}$$

= 20 days.

Question 2: Slack time in PERT analysis

- a) can never be greater than zero.
- b) is always zero for critical condition
- c) can never be less than zero.
- d) is minimum for critical events

(2016)

Solution: (d)

In PERT analysis, slack time can be

- a) +ve i.e, $S > 0$
- b) -ve i.e, $S < 0$
- c) Zero i.e, $S = 0$

But for critical events will be minimum or zero.

NOTE: Generally for critical activities slack will be zero, but it can be minimum also.

Question 3: In PERT technique, the time estimate of activities and proba-

- bility of their occurrence follow
- a) Binomial distribution
 - b) Normal distribution
 - c) Poisson distribution.
 - d) β - distribution

(2020)

Solution: (d)

In PERT analysis time estimates of activities and probability of their occurrence follow β distribution.

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CPM (Central Path Method)

This technique is developed in parallel with PERT but is difference in following ways:



PERT	CPM
- It is an event oriented. - It is based upon probabilistic approach. - Here 3 time estimates are made for each activity. - Each activity follows β-distribution. - It is used in research and development type project. - Cost of the project is directly dependent upon time of project.	- It is an activity oriented network. - Deterministic approach. - One time estimate is made for each activity. - follows normal distribution. - It is used in repetitive type project, space manufacturing industries. - Here cost is not directly dependent upon time.

CPM Analysis

$t \rightarrow$ Event time (T_E, T_L) start and finish
time of activity.

Start and finish time of Activity.



1. Earliest Start time: It is the earliest time in which an activity can be start.

— An activity can start only when its tail event occur. Hence, EST of an activity is equal to T_E of its tail event.

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2. Earliest finish time (EFT): It is the earliest time in which an activity can be start. An activity can finish in earliest time if it start at its earliest time.

$$EFT = EST + t_{ij}$$

NOTE: Earliest finish time of an activity is not equal to head event in general.

3. Latest finish time (LFT) : It is the latest time in which an activity can be finished without affecting the schedule completion time of project.

An activity can be finished latest at its latest occurrence time of head event.

$$LFT = (T_{Lj})$$



4. Latest Start time : It is the latest time in which activity can start without affecting the completion time of project.

- If an activity start at its latest time hence also finish at its latest time.

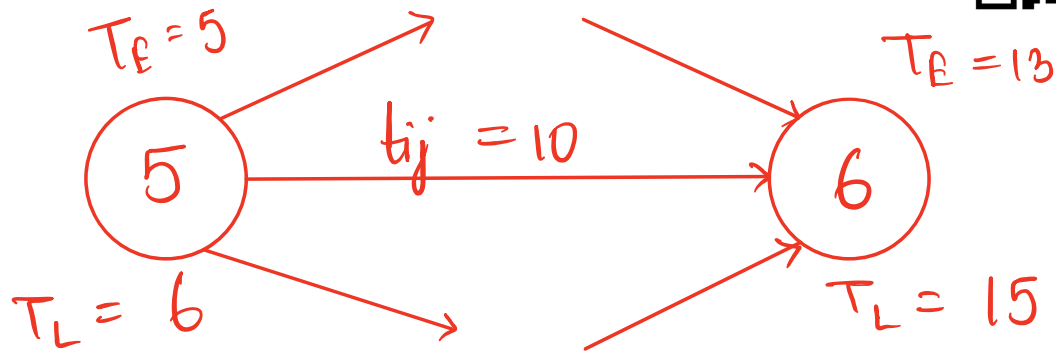
$$LST + t_{ij} = LFT$$

$$LST = LFT - t_{ij}$$

NOTE : LST of an activity is not equal to T_{Lj} (latest occurrence time of its tail event).

Activity name	EST	, EFT
T_{ij}	LST	LFT

Question : Calculate the activity line.



$$EST = 5, \quad EFT = EST + t_{ij} = 5 + 10 = 15$$

$$LFT = t_{ij} = 15, \quad LST = LFT - t_{ij} = 15 - 10 = 5$$

$$EFT - EST = LFT - LST$$

$$15 - 5 = 15 - 5$$

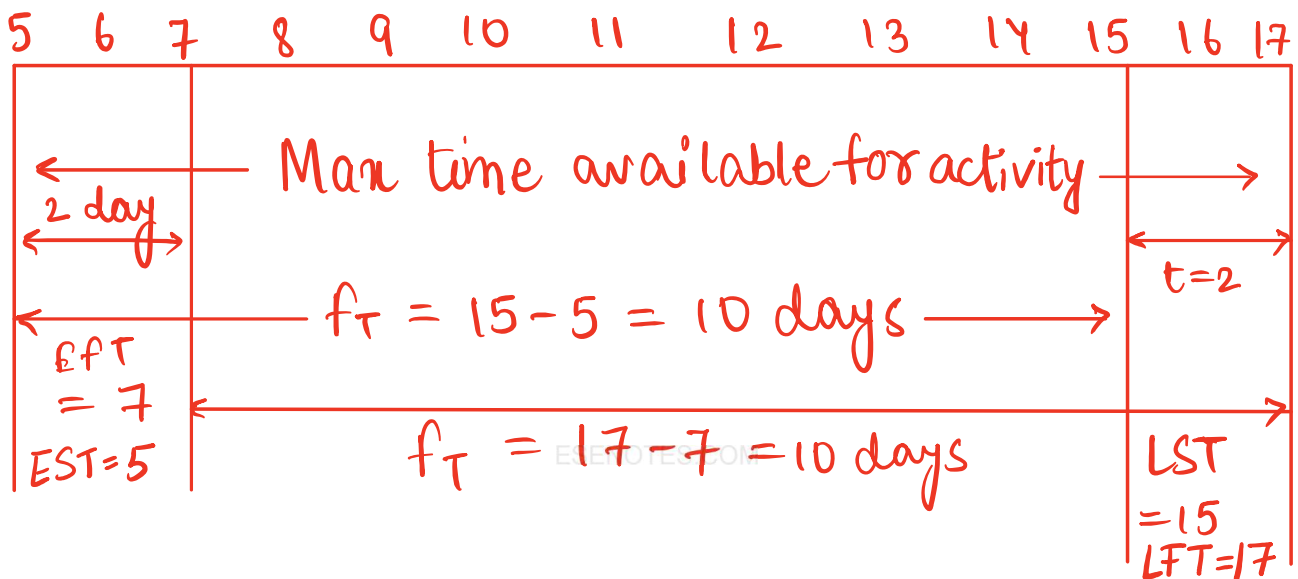
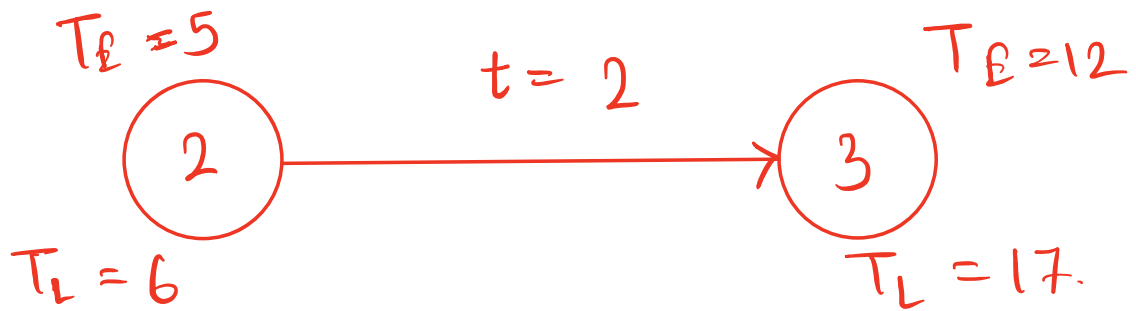


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float : It is the time by which starting and finishing of an activity can be delayed without affecting the schedule completion time of project.

- It is analogous to slack in PERT.
- floats are of following type :

1. Total float : It is the excess of max. time available over activity duration.



- Maximum time available for an activity when it starts its earliest time and finishes at latest time.

$$F_T = \{(T_L)_j - (T_E)_i\} - t_{ij}$$

$$F_T = LFT - EST - t_{ij}$$

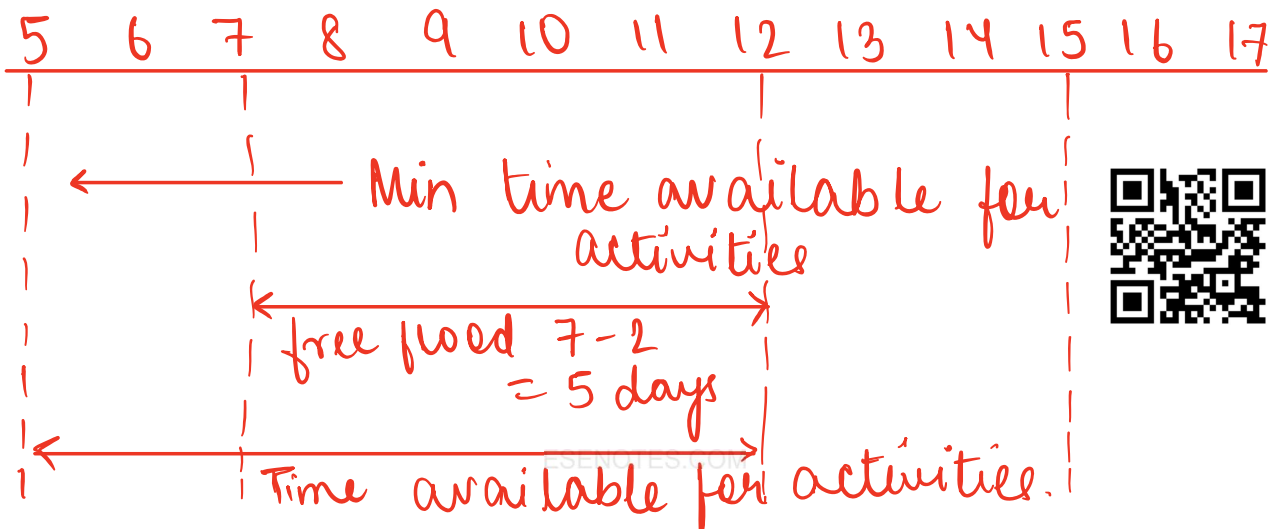
$$F_T = LST - EST$$

$$F_T = LFT - EFT$$

Total float of concern activity affects the total float of both predecessor and successor activity.



2. free float: It is the excess of time available over activity duration
- The concept of free float is on basis of fact that each event occur at its earliest time.



$$F_f = (T_{Ej} - T_{Ei}) - t_{ij} \quad \text{————— ①}$$

$$F_T = (T_i)_j - (T_E)_i - t_{ij} \quad \text{————— ②}$$

$$T_{Ei} + t_{ij} = T_{Lj} - F_T$$

$$F_f = T_{Ej} - (T_{Lj} - F_T)$$

$$= F_T - (T_{ij} - T_{Ej})$$

$$F_f = F_T - S_j = \text{Total float} - \text{Head event slack.}$$

free float of concerned activity affects the total float of predecessor activity but does not affect the total float of successor

activity.



3. Independent Float: It is the excess of min time available over the activity duration.

- An activity has minimum time for its completion when its tail event occurs at latest time and head event occurs at earliest time.

$$F_I = (T_E)_j - (T_L)_i - t_{ij}$$

$$F_F = (T_E)_j - T_{Ei} - t_{ij}$$

$$T_{Ej} - t_{ij} = F_F + T_{Ei}$$

$$F_I = F_F + T_{Ei} - T_{Li}$$

$$F_I = F_F - (T_{Li} - T_{Ei}) = F_F - S_i$$

$$F_I = F_F - \text{Tail event slack.}$$

- Independent float of concerned activity does not affect the total float of predecessor and successor activity.

Interfering float: It is another name of head event slack.

$$\begin{aligned} F_{If} &= T_{Lj} - T_{Ej} \\ &= F_T - F_F \end{aligned}$$

Analysis in CPM

To do the analysis in CPM critical path is required which is based upon critical activity that found using total float as follows.



1. If total float, $f_T < 0$: If total float is negative for an activity, it is termed as super critical activity.

- Such activity are those which runs beyond the schedule.
- Resource allocated to these activities are less than required.

2. If Total float, $f_T = 0$: If for any activity total float is zero such activity are termed as critical activity.

- These are the activity which are running on schedule.
- The resources allocated to these activities are optimum.

3. If total float, $f_T > 0$: Activities for which total float is +ve are termed as Sub-critical activities.

- There are the activities which run ahead of schedule.
- Resource allocated to these activities are more than optimum.



Critical Path : As it is the time wise longest path network in which project can be complete.

- It is the path which join critical activities.

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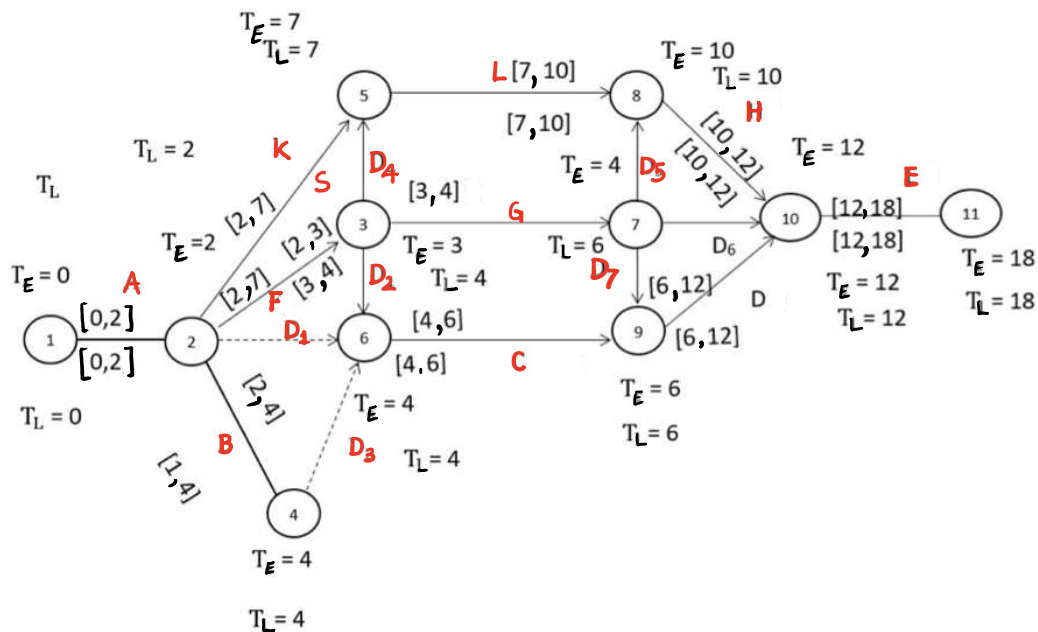
Question : Draw the network for following project and indicate event times and critical path, also find the project duration and floats for all the activities.

Activity	Duration	Depends upon
A	2	—
B	2	A
C	2	A, B, F
D	6	C, G

E	6	D, G, H
F	1	A
G	1	f
H	2	G, L
K	5	A
L	3	f, K

Activity	Duration	F_T	F_f	F_I	F_i
A	2	0	0	0	0
B	2	0	0	0	0
C	2	0	0	0	0
D	6	0	0	0	0
E	6	0	0	0	0
F	1	1	0	0	1
G	1	2	0	-1	2
H	2	0	0	0	0
K	5	0	0	0	0
L	3	0	0	0	0





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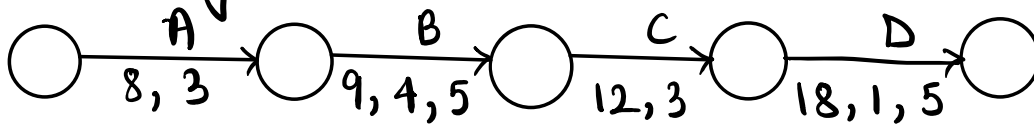
(EST, EFT) → Except f and G all
(LST, LFT) activities are critical
Critical Path: A-K-L-H-E
A-B-C-D-E



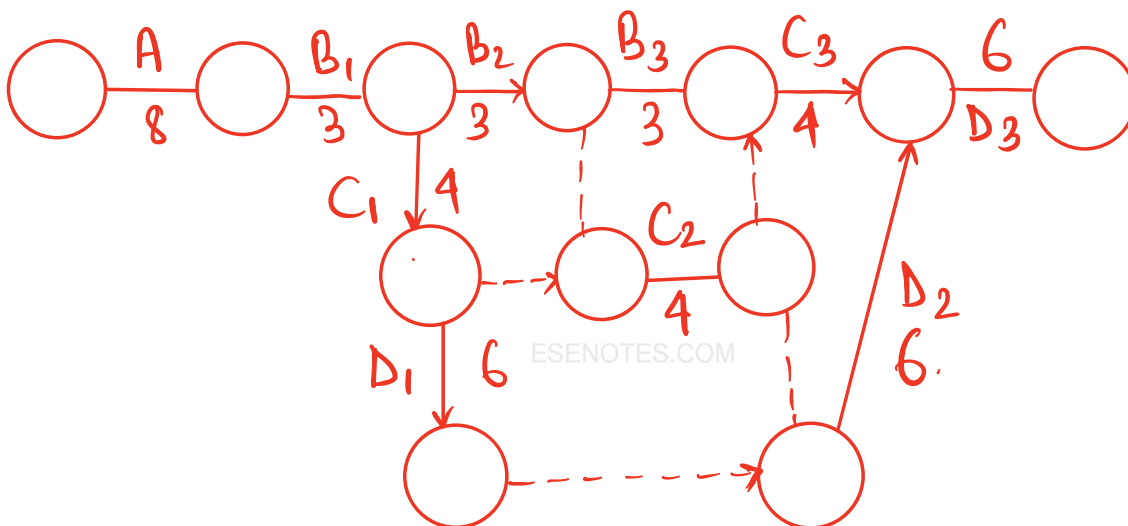
Question: With a view of improving the speed of implementation each A, B, C, D are split into 3 equal segments, maintaining appropriate interrelationship between A and each of these segments. What will be the

standard deviation of the modified project duration after segmentation.

Activity



Solution :



$$\begin{aligned}
 &A - B_1 - C_1 - D_1 - D - D_2 - D_3 \\
 \sigma_B &= \sqrt{\sigma B_1^2 + \sigma B_2^2 + \sigma B_3^2} \\
 &= \sqrt{4.5} \\
 &= 2.6
 \end{aligned}$$

$$\begin{aligned}
 \sigma_C &= 1.73 \\
 \sigma &= \sqrt{3^2 + 2.6^2 + 1.73^2 + 3(.87)^2} \\
 &= 4.6.
 \end{aligned}$$

PREVIOUS YEAR QUESTION

ESE

Question 1 : The amount of time by which the start of the activity may be delayed without interfering with the start of any succeeding activity is called

a) Activity float b) free float
c) Total float d) Interfering float.

(2019)

Solution : b)

free float is the amount of time by which the start of the activity may be delayed without interfering with the start of any succeeding activity.

Question 2 : Which of the following statements are correct for Network Critical Path?

1. The path of critical activities, which links the start and end events is critical path.
2. It is the path of activities having zero float.
3. It is the path of events having zero slack.

4. The sum of the duration of the critical activities along a critical path gives the duration of the project.

- a) 1, 2, 3 and 4 b) 1, 2 and 3 only
c) 1 and 4 only d) 2, 3 and 4 only
(2020)

Solution : c)

Critical path has activities having minimum floats.

It is not necessary that path of events having zero slack becomes critical.

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Question 3 : Consider the following activity for the total project.

Activity	Immediate Predecessors	Duration (Days)
A	—	10
B	—	9
C	A	9
D	A	8
E	B	7

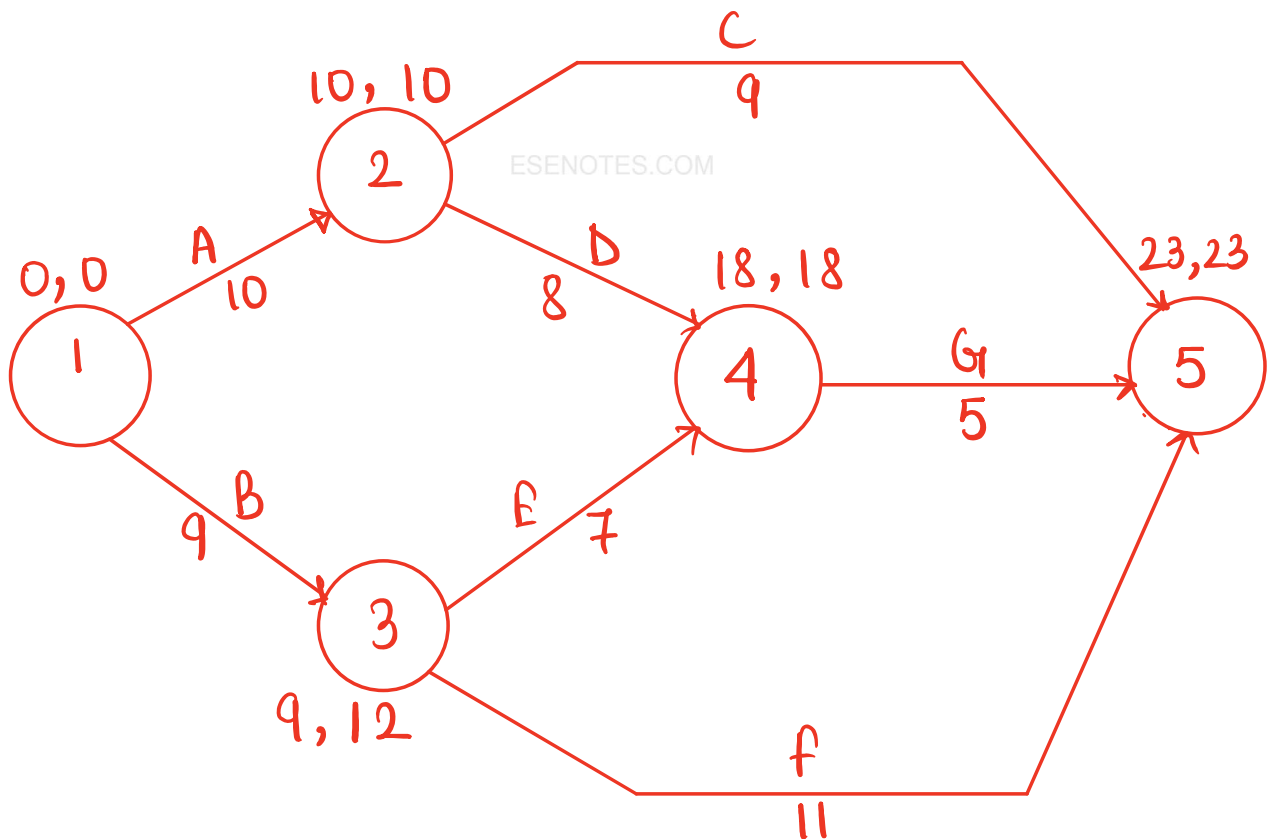
f 6	B D, E	11 5
--------	-----------	---------

The total project duration for the critical path will be

- a) 25 days
c) 21 days

- b) 23 days
d) 19 days
(2020)

Solution: b)



Project completion time = 23 days.

CRASHING

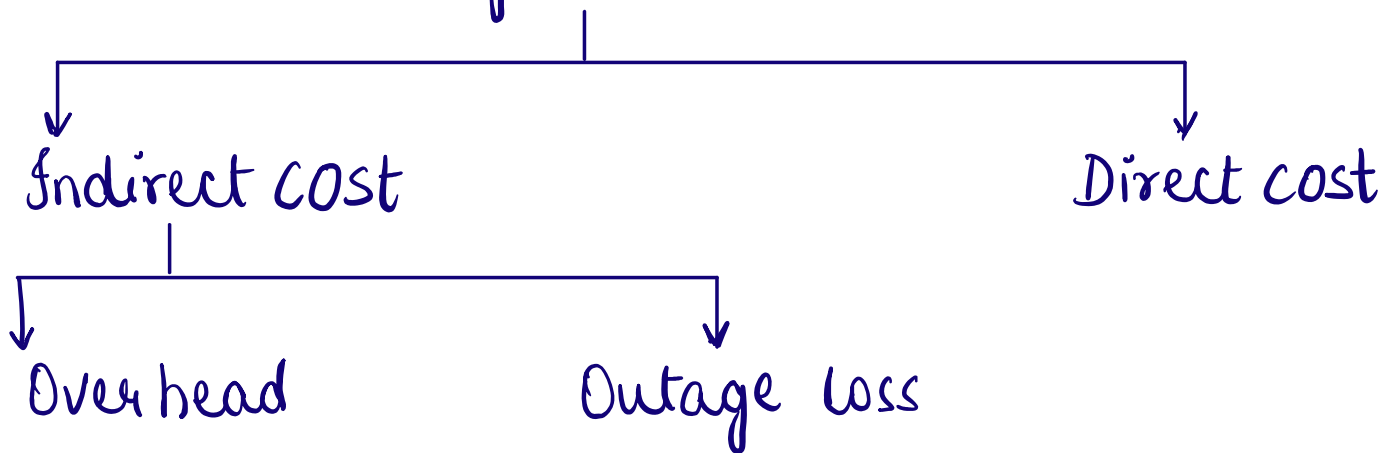


for any project, two most important elements are time and cost.

- The project cost depends on the time but not linearly and directly hence to find the min cost and optimum time project cost is to be studied.
- Total cost of the project consists of following components.

1. Indirect cost
2. Direct cost

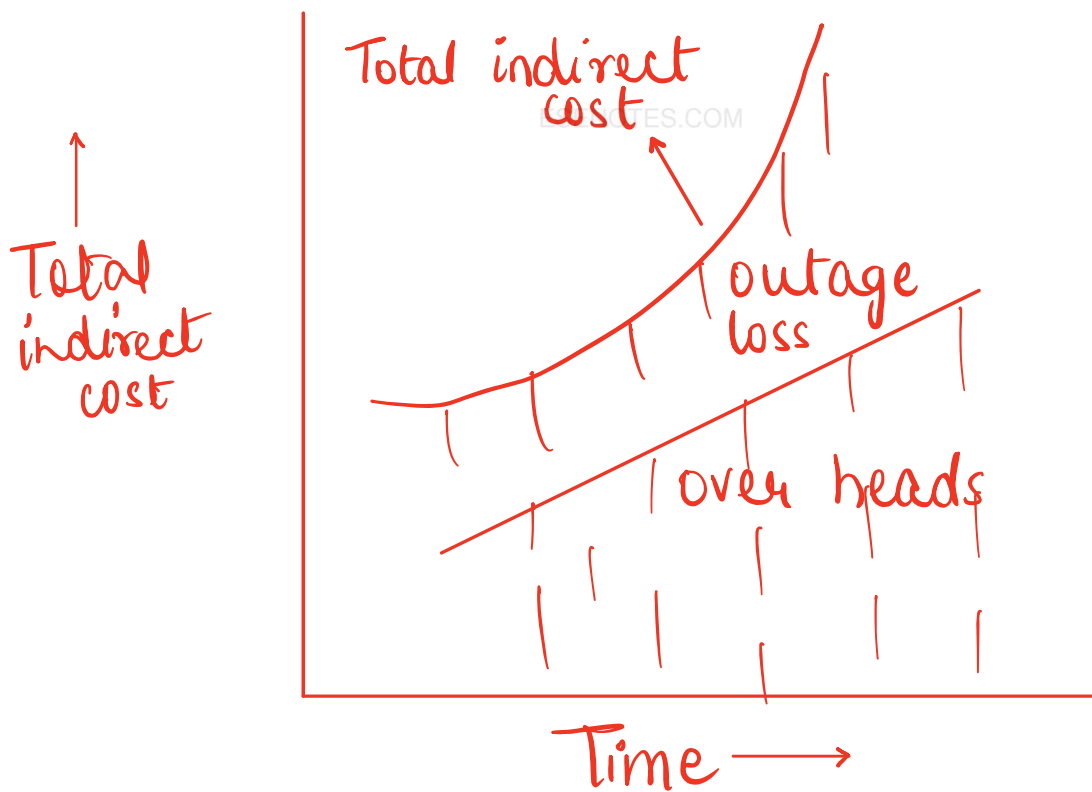
Project Cost



1. Indirect Cost: It is that cost or expenses which can not be computed individually for an activity, But can be found for the entire project.

- This cost increases directly with time.
- This is further classified as

Over heads	Outage loss
Salary given to employees (it increases linearly with time).	Penalties, taxes, establishment charges, maintenance charges (It increases non linearly with time)

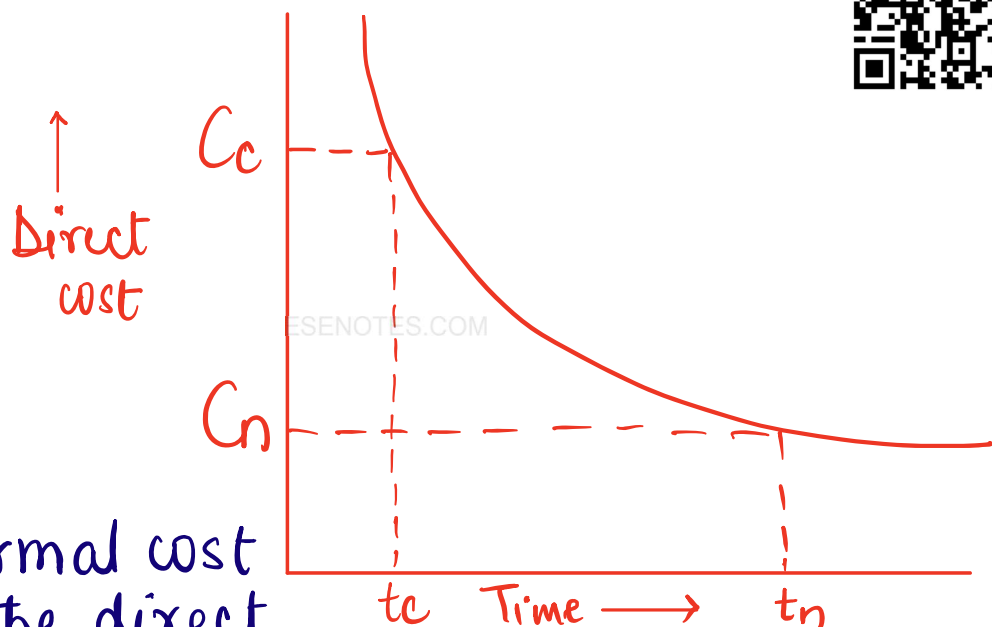


2. Direct Cost

It is the cost of project that can be controlled for each activity separately.

Ex: Machine, manpower and material.

— This direct cost decreases with increase in time.



— C_n = normal cost
It is the direct cost of an activity associated with normal time (t_n).

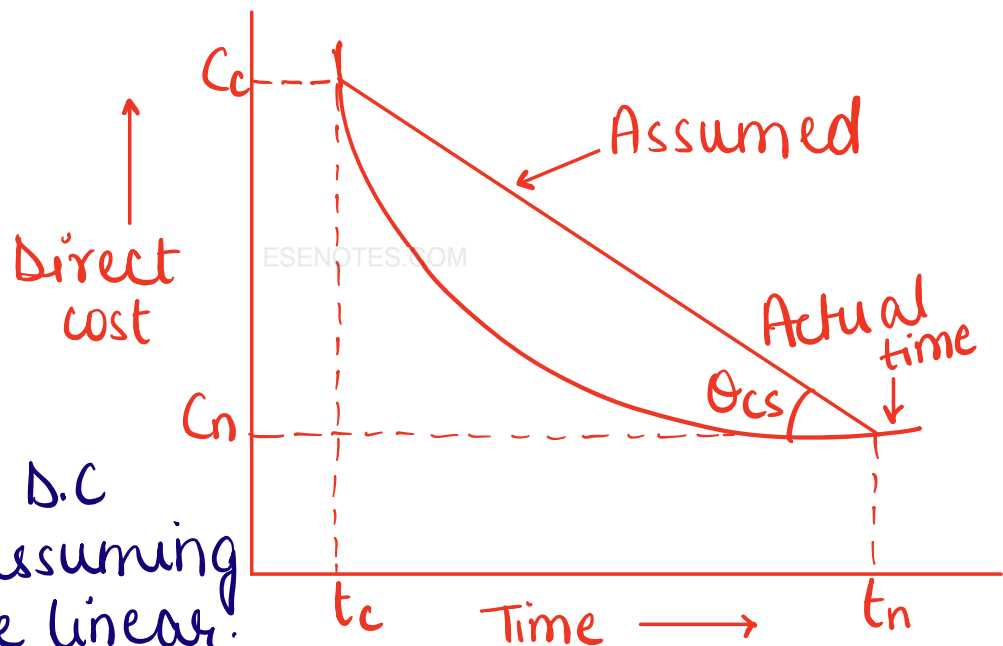
— C_c = crash cost

It is the direct cost of an activity associated with crash time.

— Crash time is min. time required for the

- completion of an activity.
- Even if we deploy infinite resources activity can not be completed in time less than crash time.
 - for simplicity in computation we assume variation of D.C to be linear with time.

Cost Slope



- Slope of D.C curve assuming it to be linear.

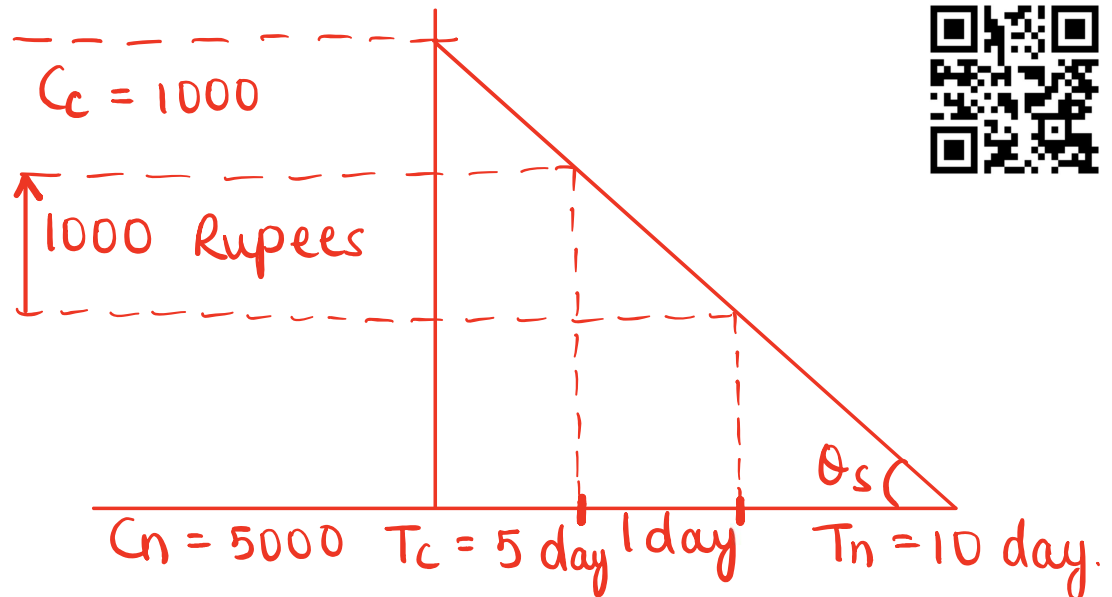
$$\theta_{cs} = \frac{C_c - C_n}{t_n - t_c}$$

$$\theta_{cs} = \frac{10000 - 5000}{10 - 5}$$

$$= 1000 \text{ Rs / day}$$

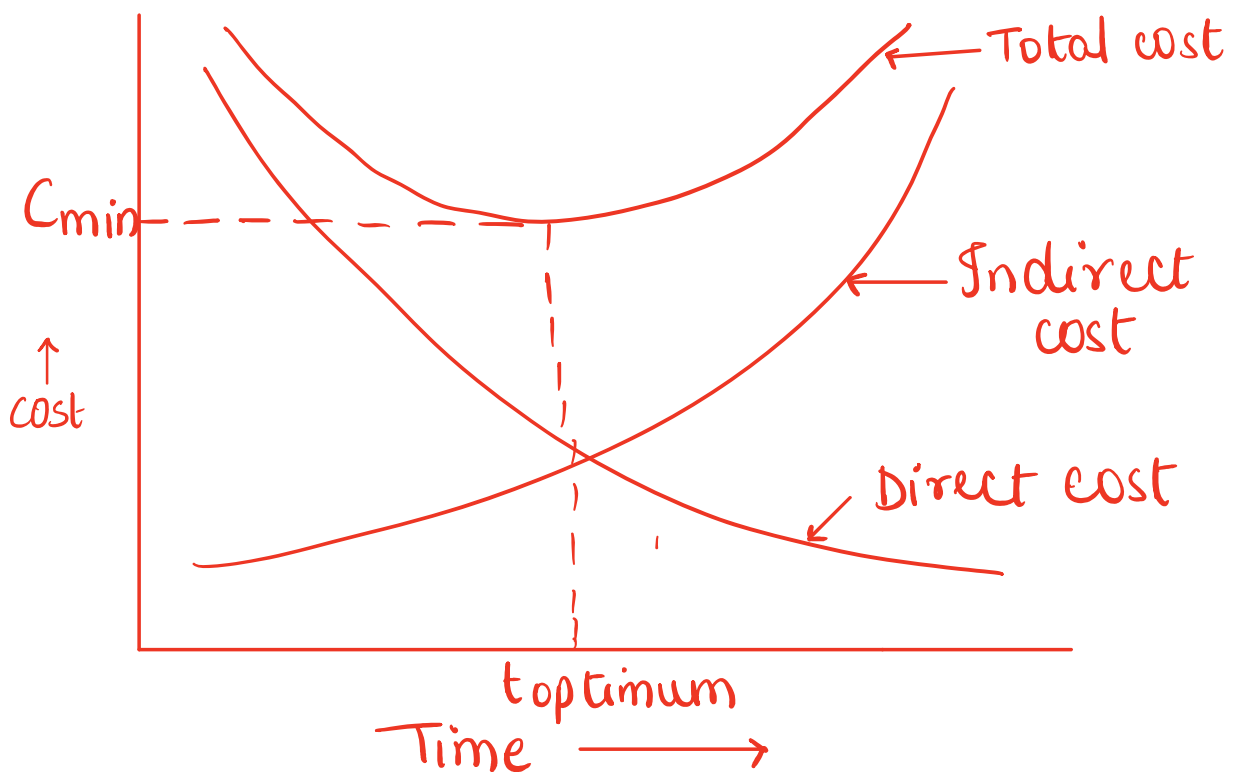
- Hence, it signifies increase in direct

cost per unit decrease in time of activity."



Total cost = Indirect cost + Direct cost.

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Crashing :

The process of reduction of total project duration along critical path to obtain min. cost is termed as Crashing.



- It being carried out as follows:
 - a) Draw the network diagram.
 - b) Perform the analysis to find critical path.
 - c) Indicate the critical path along the network (time scale).
 - d) Find the cost slope of all activity.
 - e) Start crashing the network along the critical path starting with those having least cross-slope.
 - f) While doing so, some non-critical activity may also become critical leading to the parallel critical path, In such case || crashing is done upto an extent activity are fully crashed.

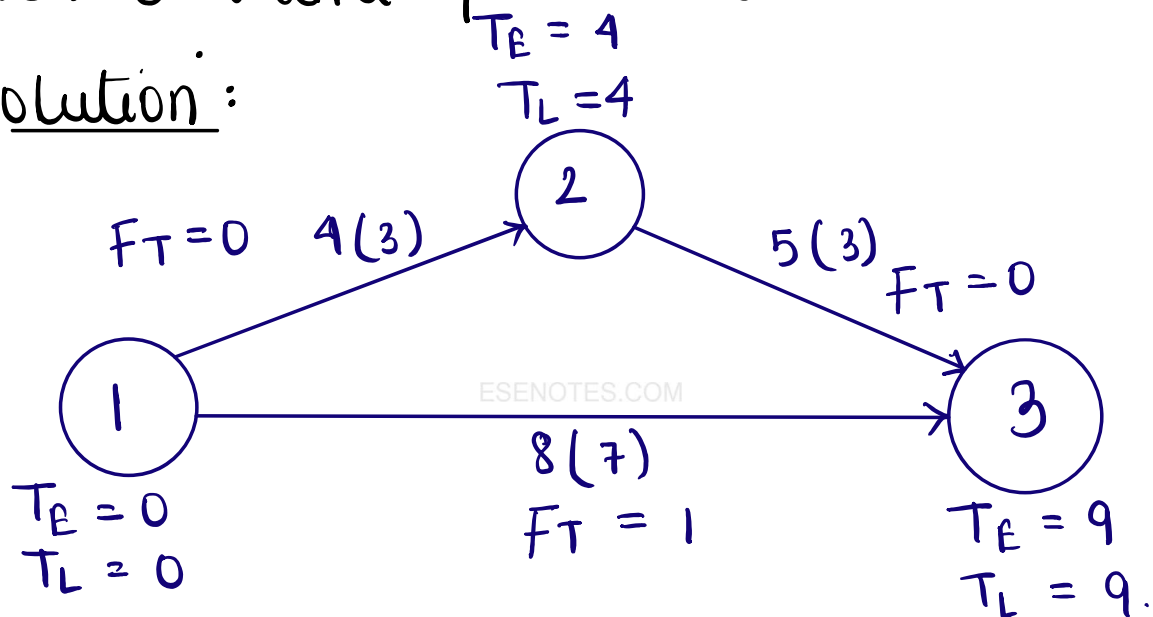
Question : The following information is available about various activities.

Activity	t_n	C_n	t_c	C_c
1-2	4	4000	3	7000
1-3	8	5000	7	8000
2-3	5	8000	3	10000



Consider overhead cost = 2000 Rs. Determine min. cost and optimum time.

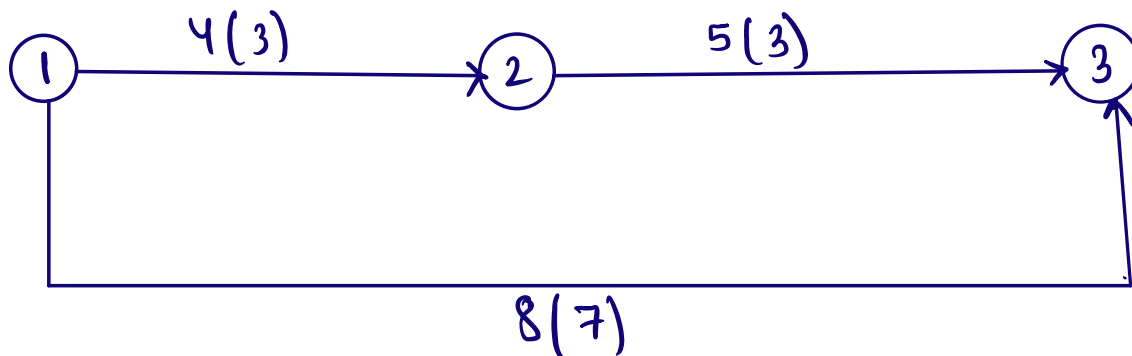
Solution:



① $t_n = 9$ week

$$OVC = 2000 \times 9 = 18000.$$

0 1 2 3 4 5 6 7 8 9



C.S	1-2	3000
	1-3	3000
	2-3	1000



$$D.C = 4000 + 5000 + 8000 \\ = 17000 \text{ Rs}$$

$$\text{Total cost} = 18000 + 17000 \\ = 35000 \text{ Rs}$$

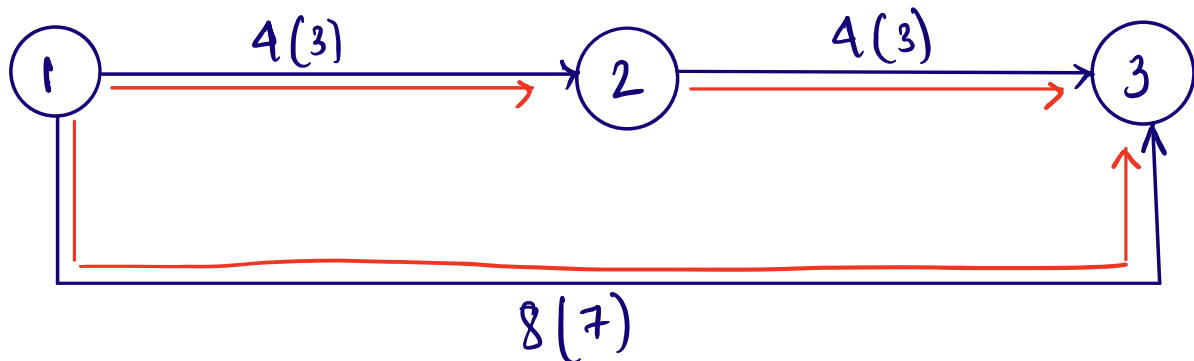
2. First Stage Crashing.

2-3 at 1 week

$$T_c = 8 \text{ week}$$

0 1 2 3 4 5 6 7 8 9

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$$T_c = 8 \text{ week.}$$

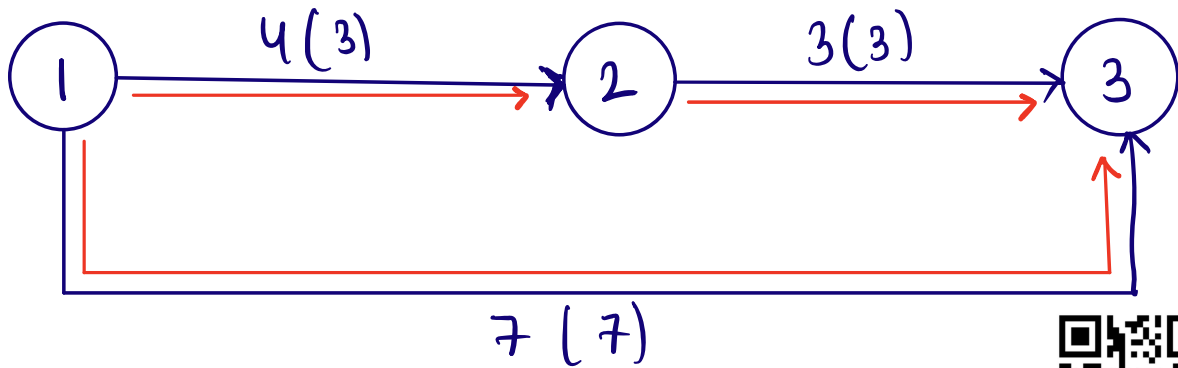
$$D.C = 16000 \text{ Rs}$$

$$DC = 17000 + 1000 \times 1 \\ = 18000 \text{ Rs}$$

$$\text{Total cost} = 34000 \text{ Rs.}$$

3. Second stage crashing

		2-3	11	1-3	@ 1 week		
0	1	2	3	4	5	6	7

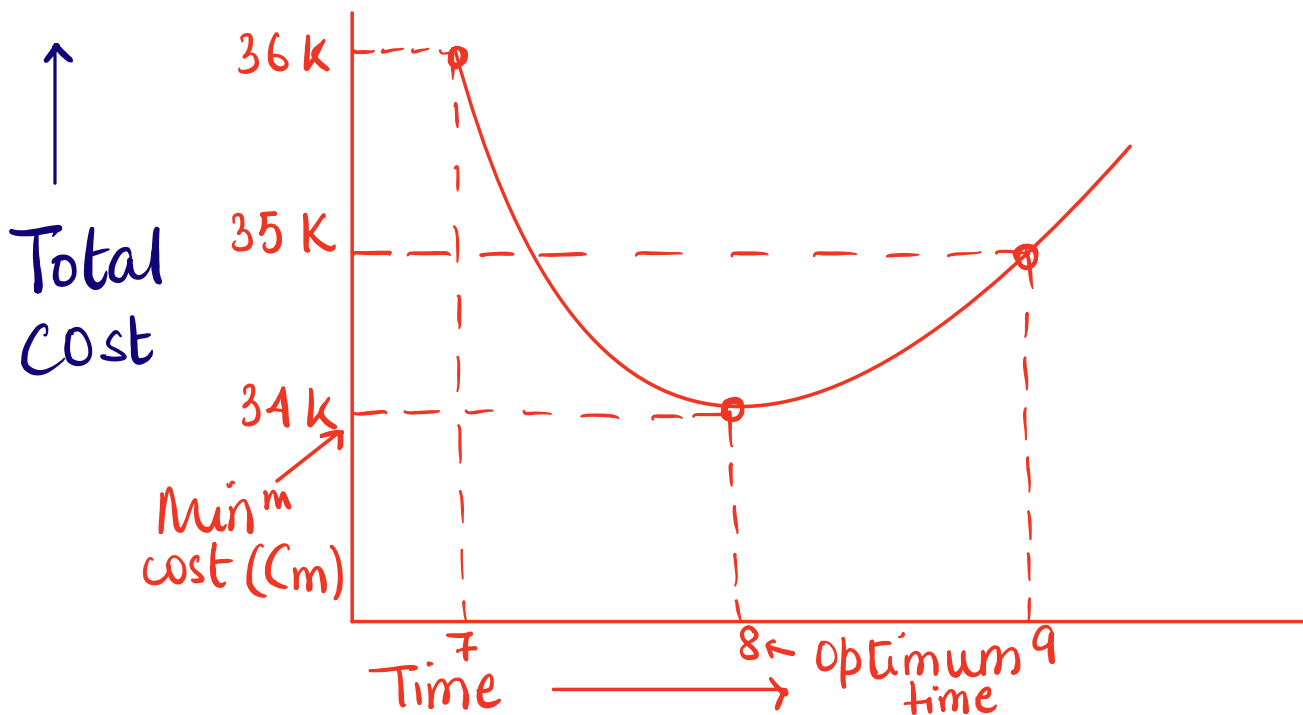


$$T_c = 7 \text{ week}$$

$$IDC = 7 \times 2000 = 14000 \text{ Rs}$$

$$DC = 18000 + (3000 + 1000) = 22000 \text{ Rs}$$

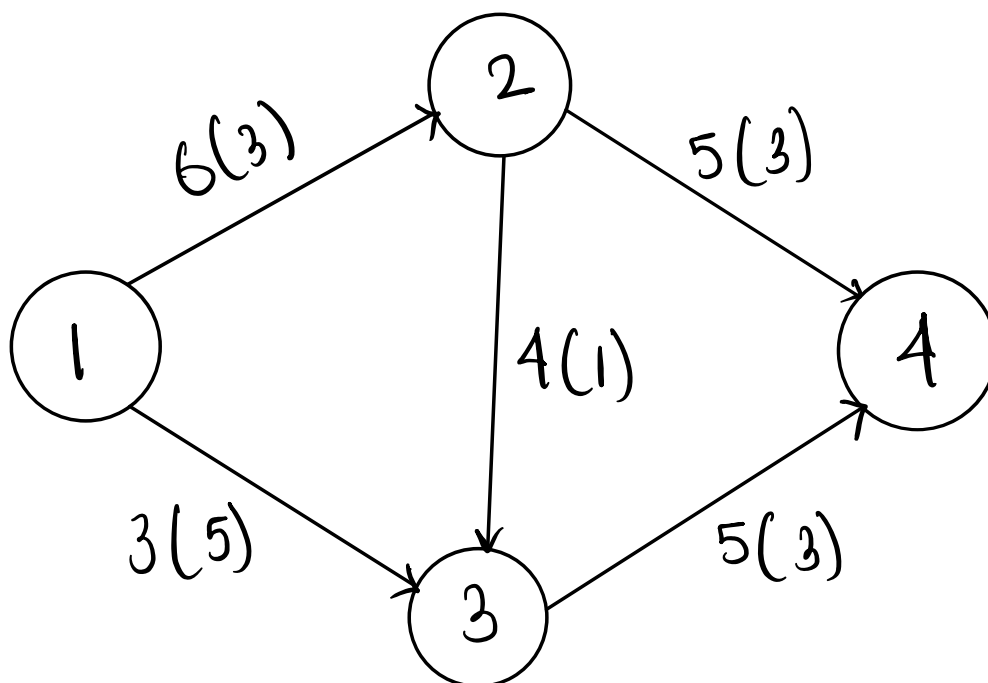
$$\text{Total cost} = 36000 \text{ Rs.}$$



Question: The following table gives the data for the duration and cost for each activity of a project network shown in fig. The indirect cost of project is Rs 3000/week. Determine the optimum time and minimum cost of project.

Activity	t_n	C_n	t_c	C_c	$C.s.$
1-2 *	6	7000	3	14500	2500
1-3	8	4000	5	8500	1500
2-3 *	4	6000	1	4000	1000
2-4	5	8000	3	15000	3500
3-4 *	5	5000	3	11000	3000

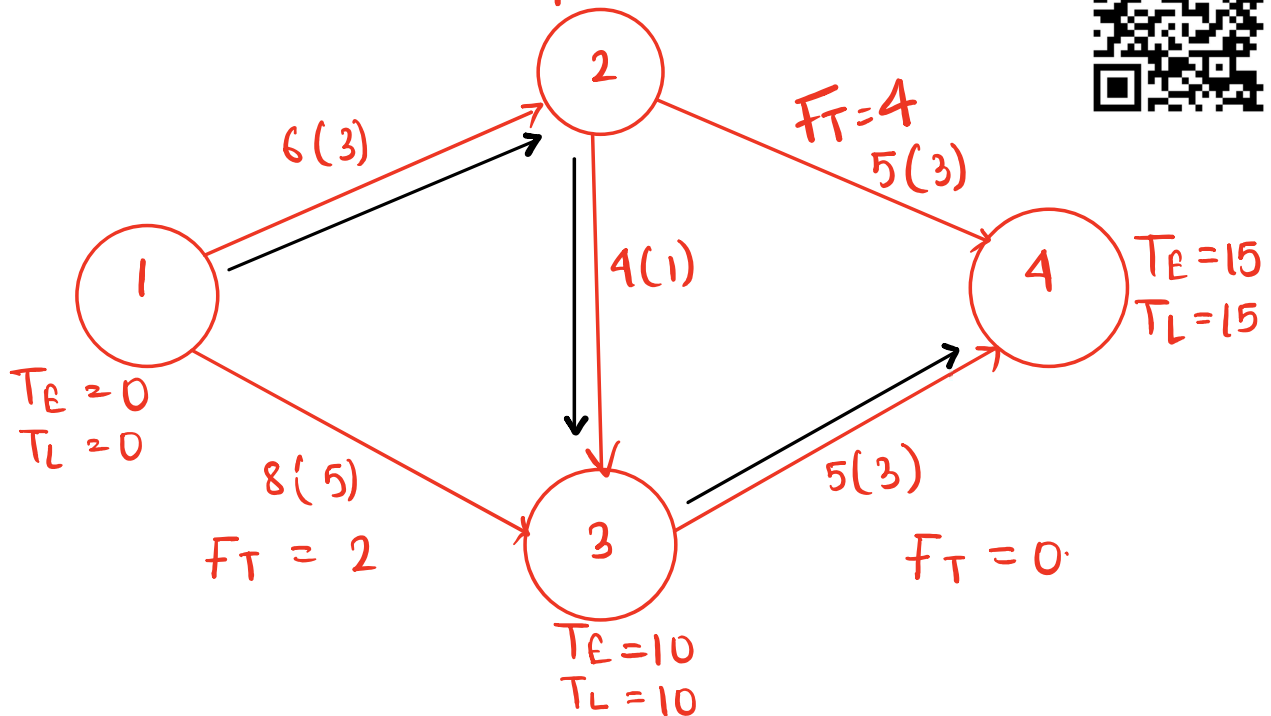
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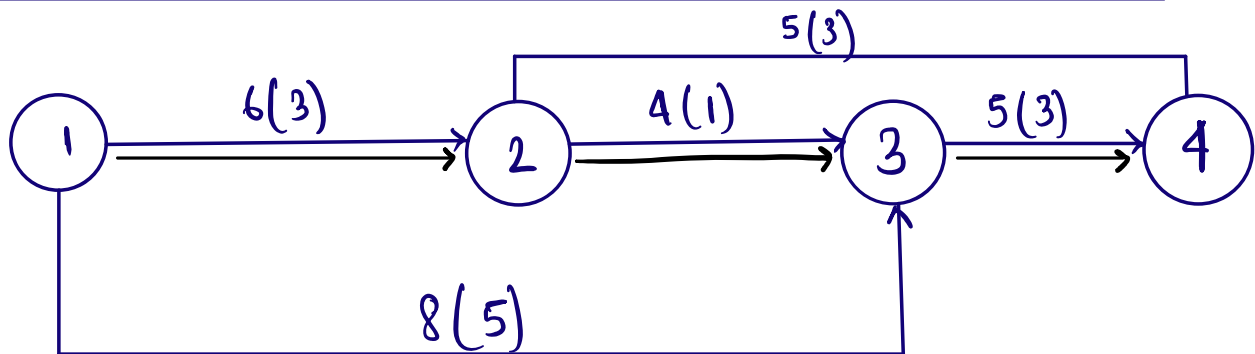
Solution :

$$T_E = 6, T_L = 6.$$

$$F_T = 0$$



0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15



$$t_n = 15 \text{ days}$$

$$IDC = 3000 \times 15$$

$$= 45000 \text{ Rs}$$

$$D_C = 7000 + 6000 + 5000 + 4000 + 8000$$

$$= 30000$$

Total cost = 750000.



1. First stage Crashing

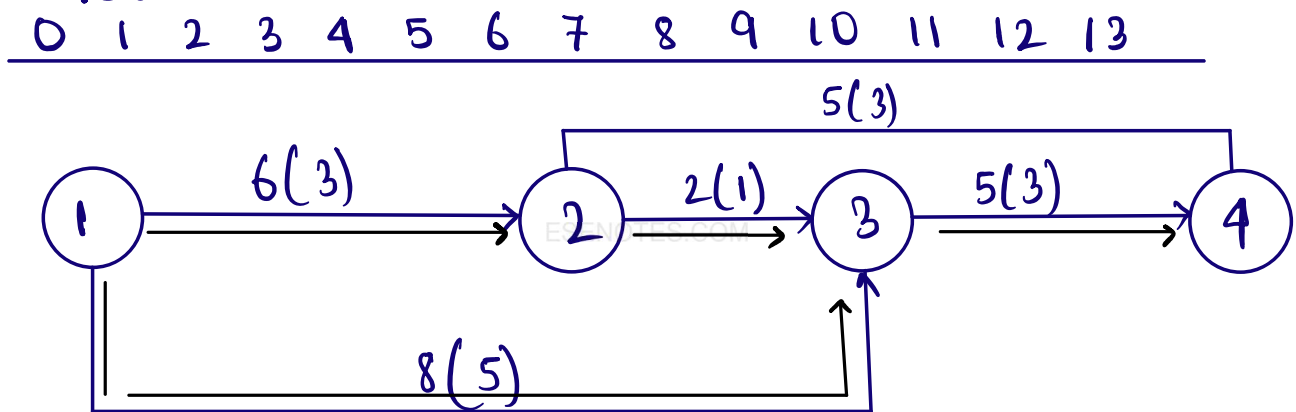
2-3 at 2 week

$T_c = 13$ week

$2DC = 3000 \times 13 = 39000$ Rs

$DC = 30000 + 1000 \times 2$
 $= 32000$ Rs

Total cost = 71000.



2. Second stage crashing

2-3 11 1-3 2.5 k

1-2 11 1-3 4 k

3-4 3 k.

Crashing of 2-3 11 1-3 at 1 week

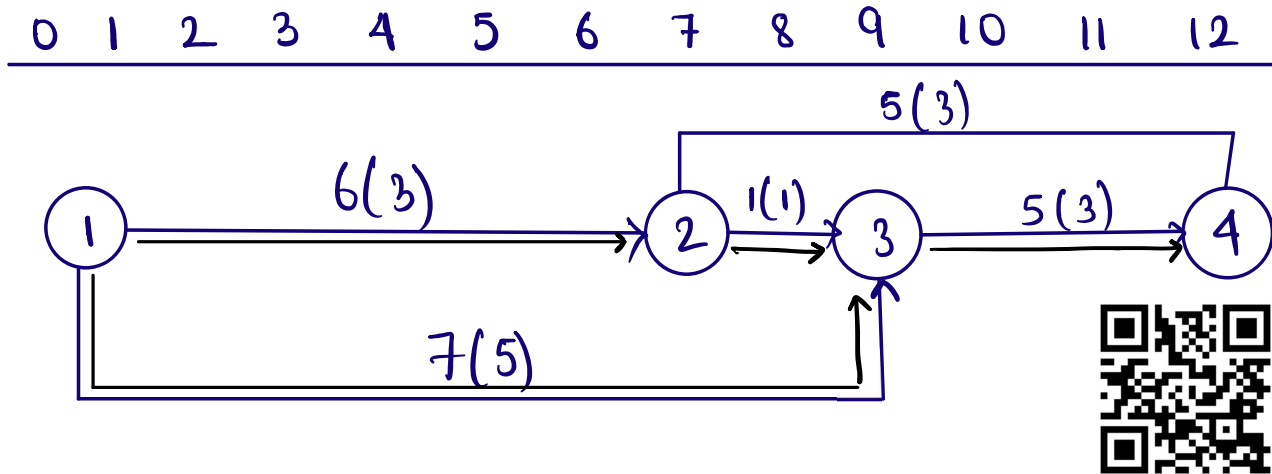
$T_c = 12$ week.

$2DC = 3000 \times 12 = 36000$ Rs

$DC = 32000 + 2500$

$$= 34500$$

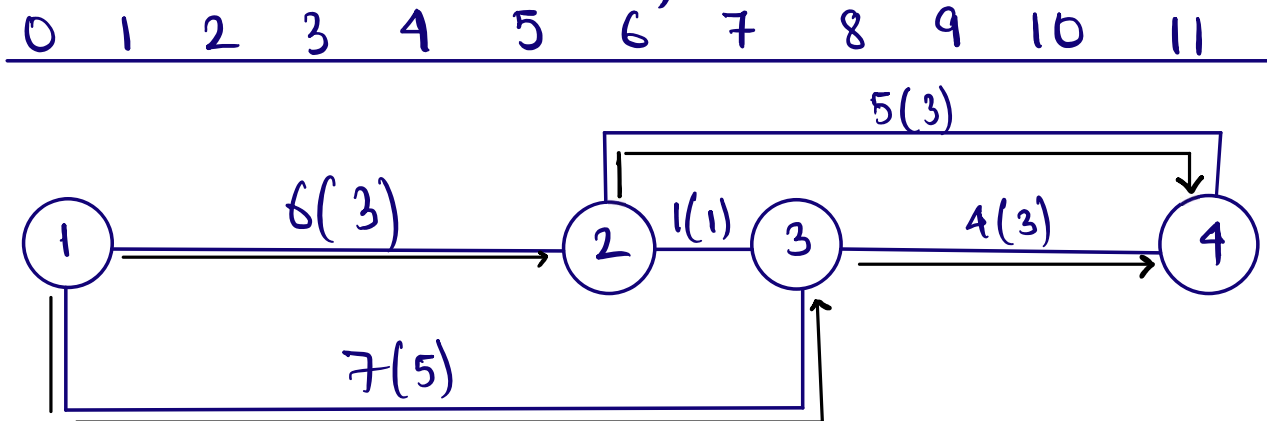
$$\text{Total cost} = 70500$$



3. Third stage crashing

1-2 11 1-3
2-3 11 1-3 X
3-4 ✓

3-4 at 1 week, $T_c = 11$ weeks



$$\text{IDC} = 3000 \times 11$$

$$= 33000$$

$$\text{DC} = 34500 + 3000 \times 1$$

$$= 37500$$

$$TC = 70500 \text{ Rs.}$$



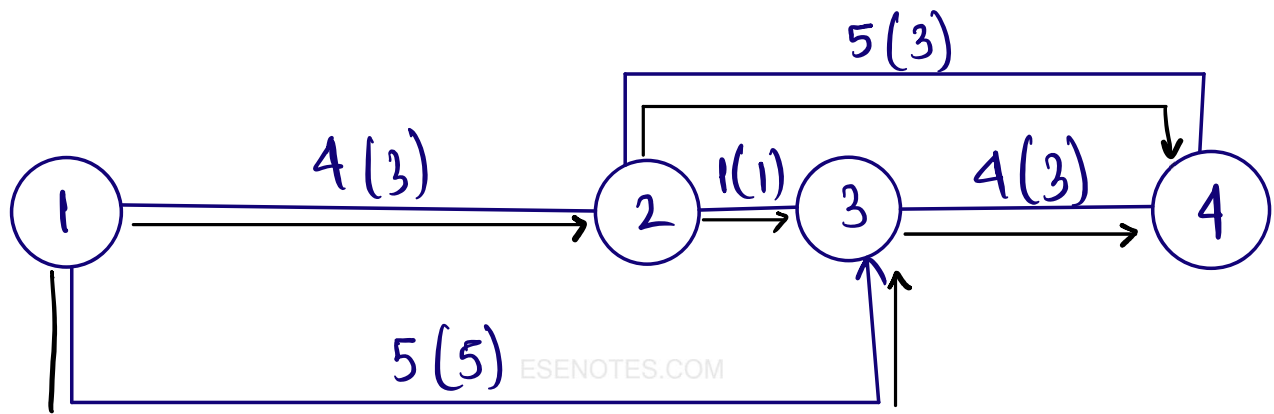
4. fourth stage crashing

1-2 || 1-3 4k ✓

2-4 || 3-4 6.5k

1-2 || 1-3 at 2 week, $T_c = 9$ weeks

0 1 2 3 4 5 6 7 8 9



$$IC = 3000 \times 9 = 27600$$

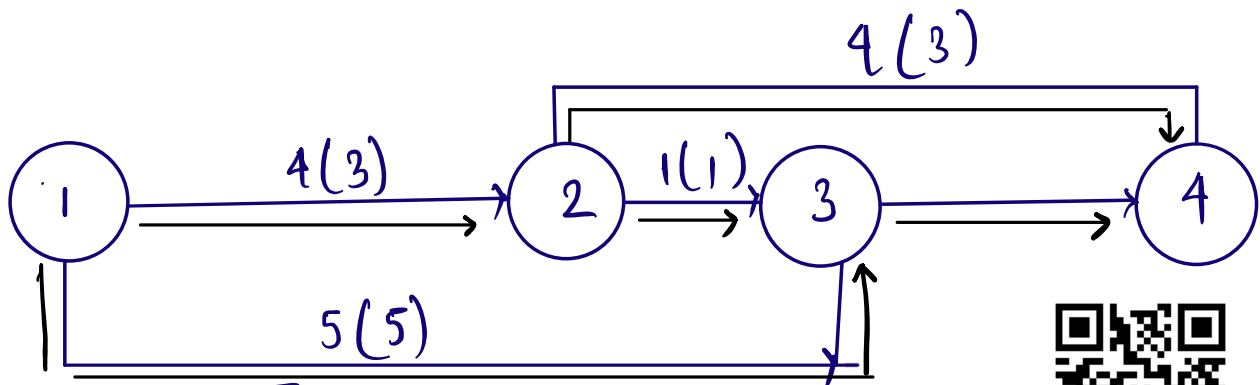
$$DC = 37500 + 4000 \times 2 = 45500$$

$$TC = 72500$$

5. fifth stage

2-4 || 3-4 at 1 week.

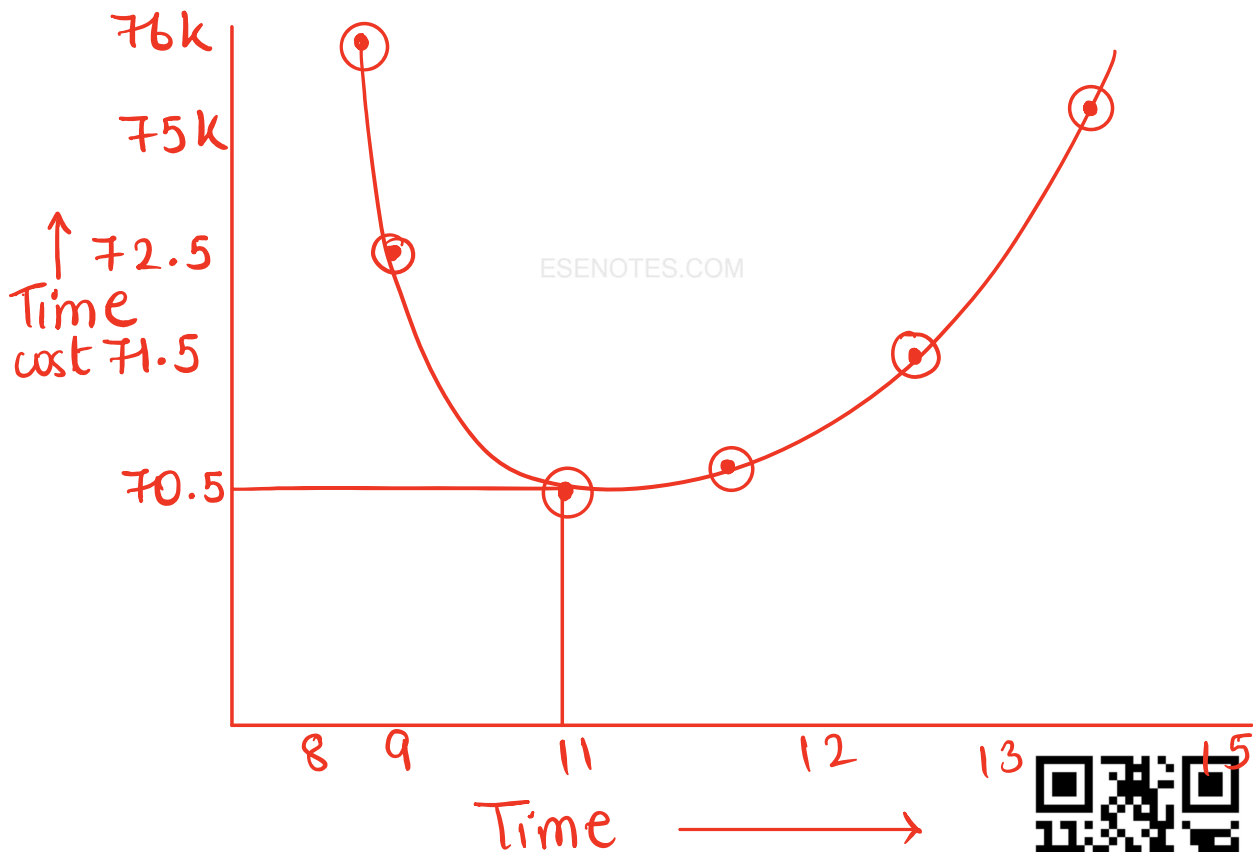
0 1 2 3 4 5 6 7 8



$$T_c = 24000$$

$$D_c = 52000 \text{ Rs}$$

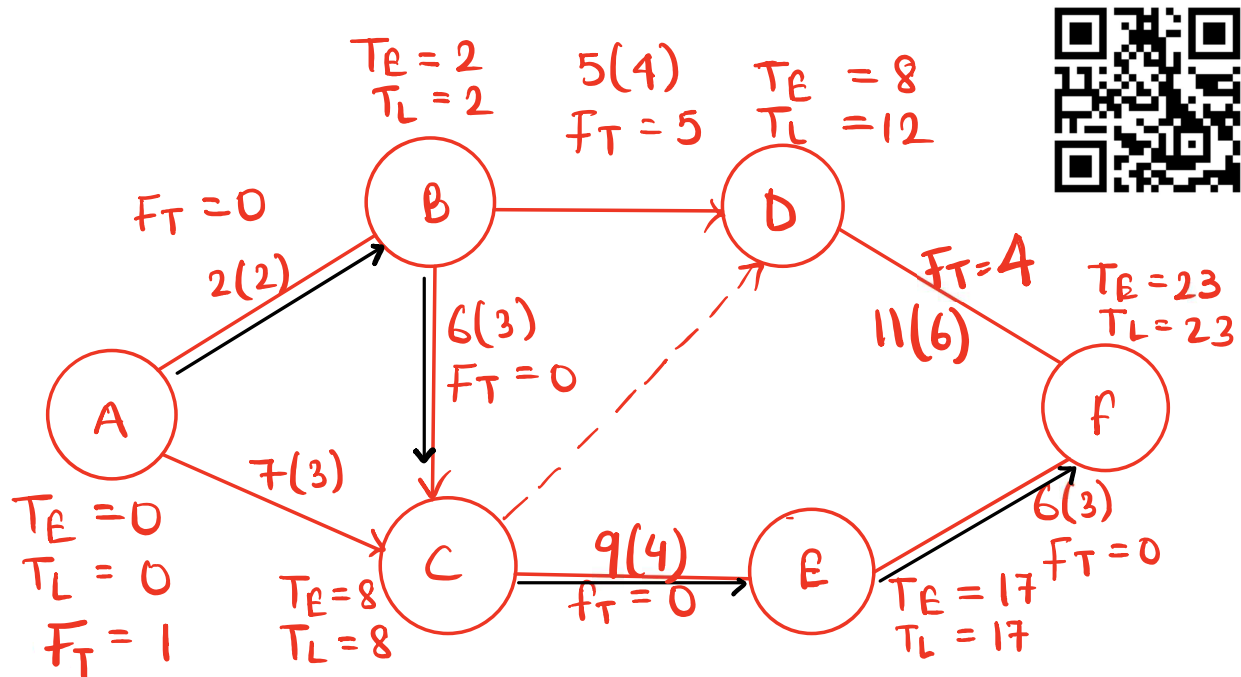
$$T_c = 76000 \text{ Rs}$$



Question : Determine the minimum cost and optimum duration for

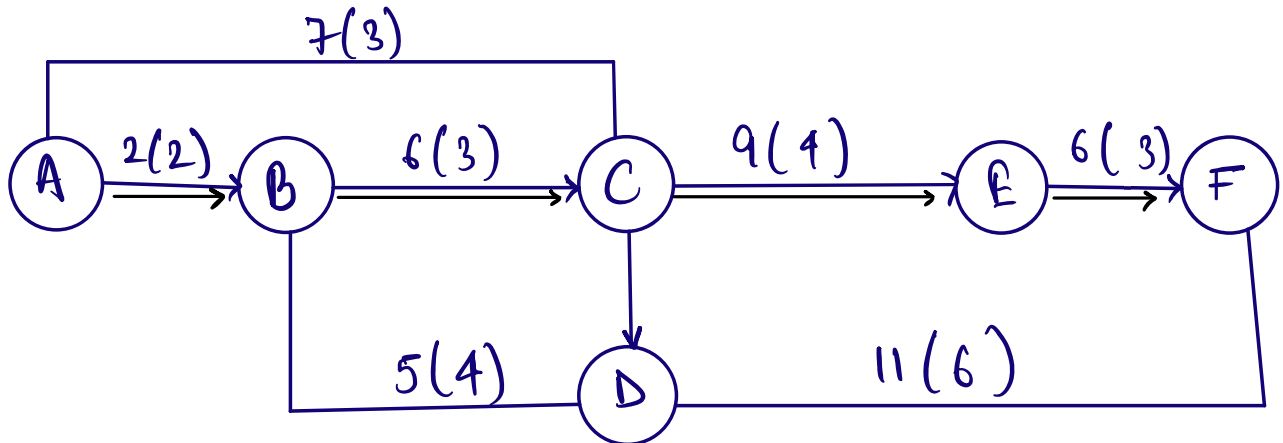
project network. The duration and cost of each activity of the project is given in the following table the indirect cost of project is Rs 800/day.

Activity	t_n (days)	C_n	t_c (days)	C_c (Rs)	C_s Rs/day
A-B	2	10000	2	10000	0
A-C	7	5000	3	9000	1000
B-C	6	3000	3	4200	400
B-D	5	2000	4	2500	500
C-D	0	0	0	0	0
C-E	9	6000	4	9000	600
D-f	11	6000	6	10000	800
E-f	6	7000	3	9000	700



A - B - C - E - F Critical Path.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17
18 19 20 21 22 23



$t_n = 23$ days

$IDC = 800 \times 23$

$= 18400$ Rs

$D_c = 39000$ Rs

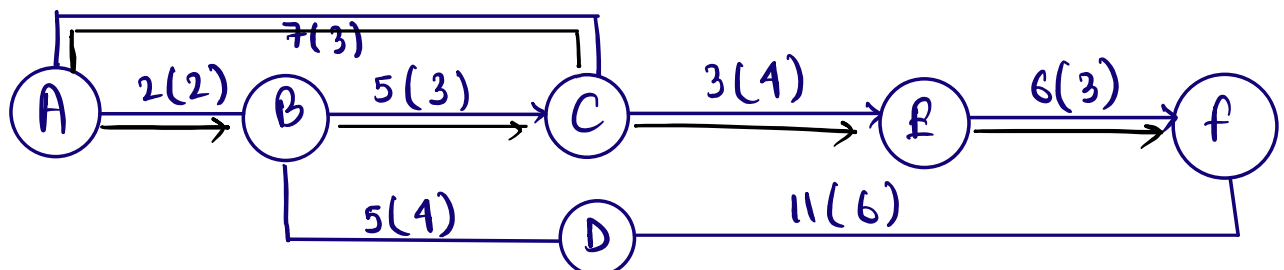
$T_c = 57400$ Rs.



First Stage Crashing

BC at 1 week.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
17 18 19 20 21 22



$$\begin{aligned}
 t_c &= 22 \text{ days} \\
 LDC &= 17600 \text{ Rs} \\
 DC &= 39000 + 400 \\
 &= 39400 \text{ Rs} \\
 TC &= 57000 \text{ Rs}
 \end{aligned}$$

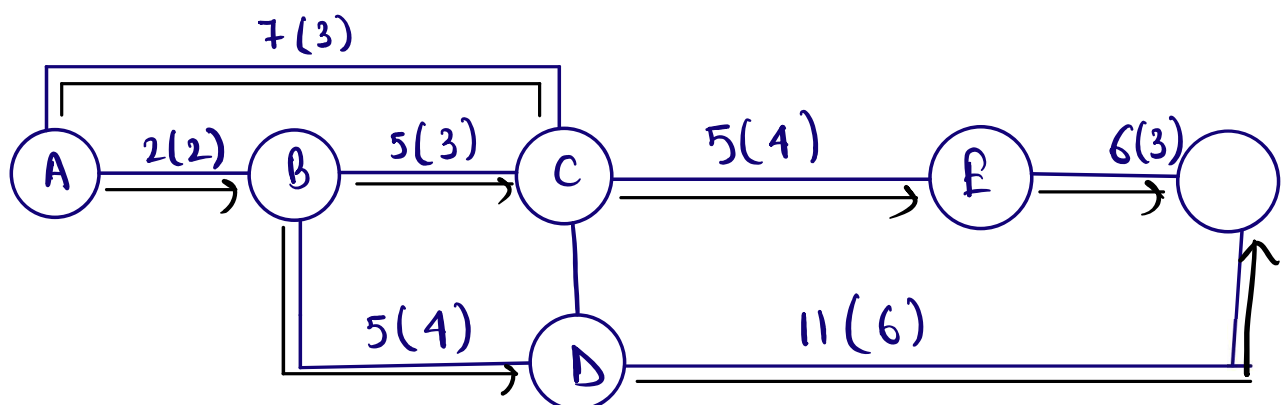


Second stage Crashing

CE at 4 days	AC BC	CS
$T_c = 13 \text{ weeks}$	CE	1400
$LDC = 800 \times 18$	EF	600
$= 14400 \text{ Rs}$		700
$DC = 39400 + 600 \times 4$		
$= 41800 \text{ Rs}$		

Total cost = 56200 Rs.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
16 17 18

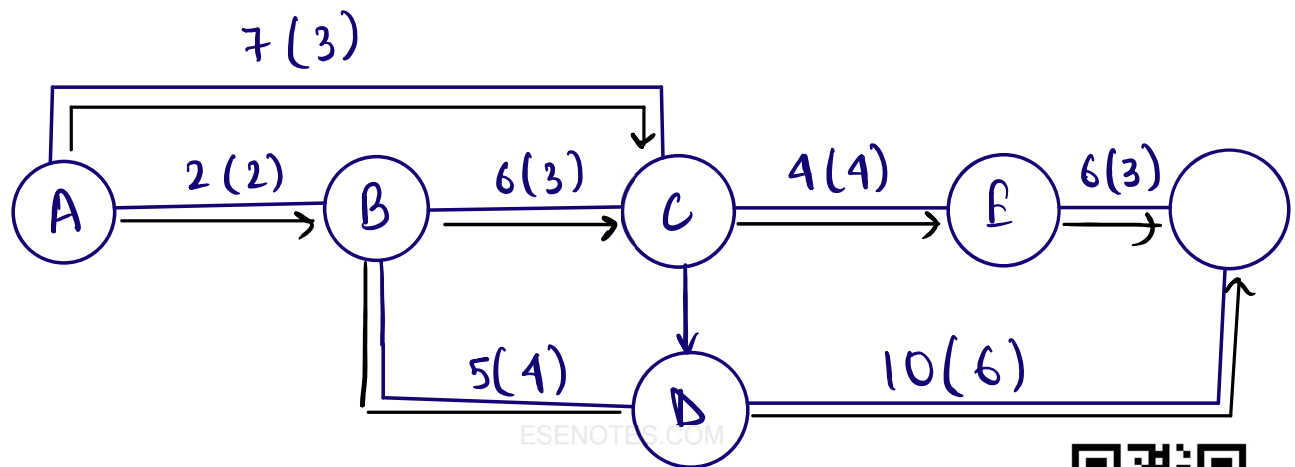


Third stage crashing

CE || DF at 1 days

	CS
AC BC BD	1900
CE DF	1400
Ef Df	1500

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
16 17



$$t_c = 17 \text{ days}$$

$$IDC = 17 \times 800$$

$$= 13600 \text{ Rs}$$

$$DC = 41800 + 1400 \times 1 = 43200 \text{ Rs}$$

$$\text{Total cost} = 56800 \text{ Rs.}$$

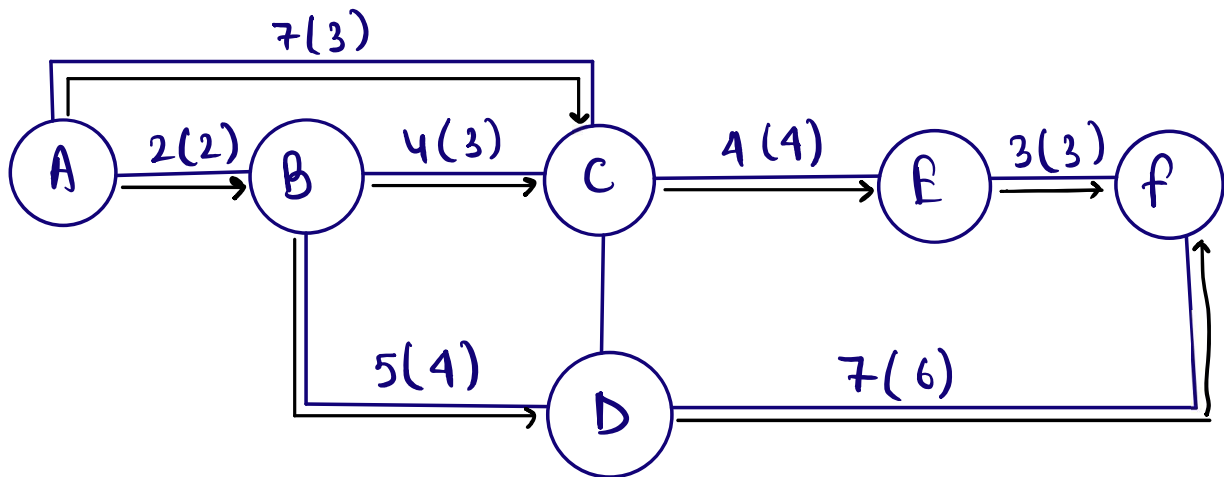


Fourth Stage crashing

Ef || Df at 3 days

	CS
AC BC BD	1900
Ef Df	1500

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14



$$t_c = 14 \text{ days}$$

$$PDC = 300 \times 14$$

$$= 11200 \text{ Rs}$$

$$DC = 43200 + 1500 \times 3$$

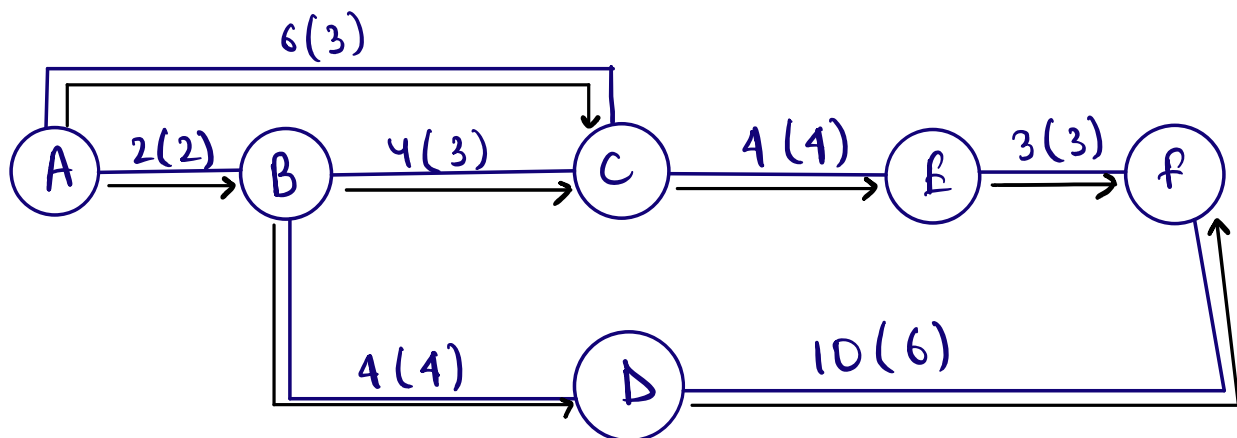
$$= 47700 \text{ Rs}$$

$$\text{Total cost} = 58900 \text{ Rs.}$$

Fifth stage crashing

AC || BC || BD at 1 days.

0 1 2 3 4 5 6 7 8 9 10 11 12 13



$$T_c = 13 \text{ days}$$

$$PDC = 13 \times 800$$

$$= 10400 \text{ Rs}$$

$$DC = 47700 + 1900$$

$$= 49600 \text{ Rs}$$

$$\text{Total cost} = 60000 \text{ Rs}$$

$$T_o = 18 \text{ days}$$

$$T_{\min} = 56200 \text{ Rs.}$$



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PREVIOUS YEAR QUESTION

ESE

Question 1: Which of the following relations are correct for determining different components of a bid price?

1. Bid price = Direct cost + Indirect cost + Mark up amount.
 2. Direct cost = Project overheads + Common plant and equipment cost + Common work men cost
 3. Mark up amount = Profit + Contingency + Allowances for risks + General overheads
- a) 1, 2 and 3 b) 1 and 2 only
c) 1 and 3 only d) 2 and 3 only.
- (2020)

Solution: c)

Bid price = Base cost + Mark up amount.

Base cost = Direct cost + Indirect cost.

Direct cost = Labour cost + Plant and equipment cost + material cost + Sub-contractor cost.

Mark up cost = Profit + Contingency + General overheads + Allowance for risks.

Question 2: Indirect cost due to accidents includes

- a) Legal charges
- b) Medical expenses for the injured.
- c) Compensation amount to the injured.
- d) Over time payment to make up the loss of time.

(2020)

Solution: d)

Cost of an accident = Direct cost + Indirect cost.

Direct cost include worker's compensation payment, medical expenses and legal services.

Indirect cost includes

- Training replacement employees.
- Accident investigations.
- Lost productivity
- Cost of absenteeism.
- Implementation of corrective measure.

Question 3: In a construction project, the cost slope of an activity is an

- indication of
- a) extra time needed
 - b) extra cost needed
 - c) reduction of duration of critical activity.
 - d) crashing of an activity.
- (2013)

Solutions : b)

Cost slope of an activity indicates increase in cost if duration of activity is reduced by one day.

$$\text{Cost slope} = \frac{\text{Crash cost} - \text{Normal cost}}{\text{Normal time} - \text{Crash time}}.$$

Updating



During the execution of the project, changes may be observed in the duration of activities i.e, it may be running ahead of schedule or behind schedule.

- The process of incorporating these changes in network is termed as updating.
- It is done whenever there is major change in activity.
- It is done when there is change in critical activity.
- No activity is added or deleted during updating.
- Critical path and critical activities and project duration may change during updating.
- For smaller project updating is more frequent than bigger project.

Data required for updating is as follows:

- A. Original network.
- B. Time computation.

- c. Time at which updating is done.
- d. New information about activity.

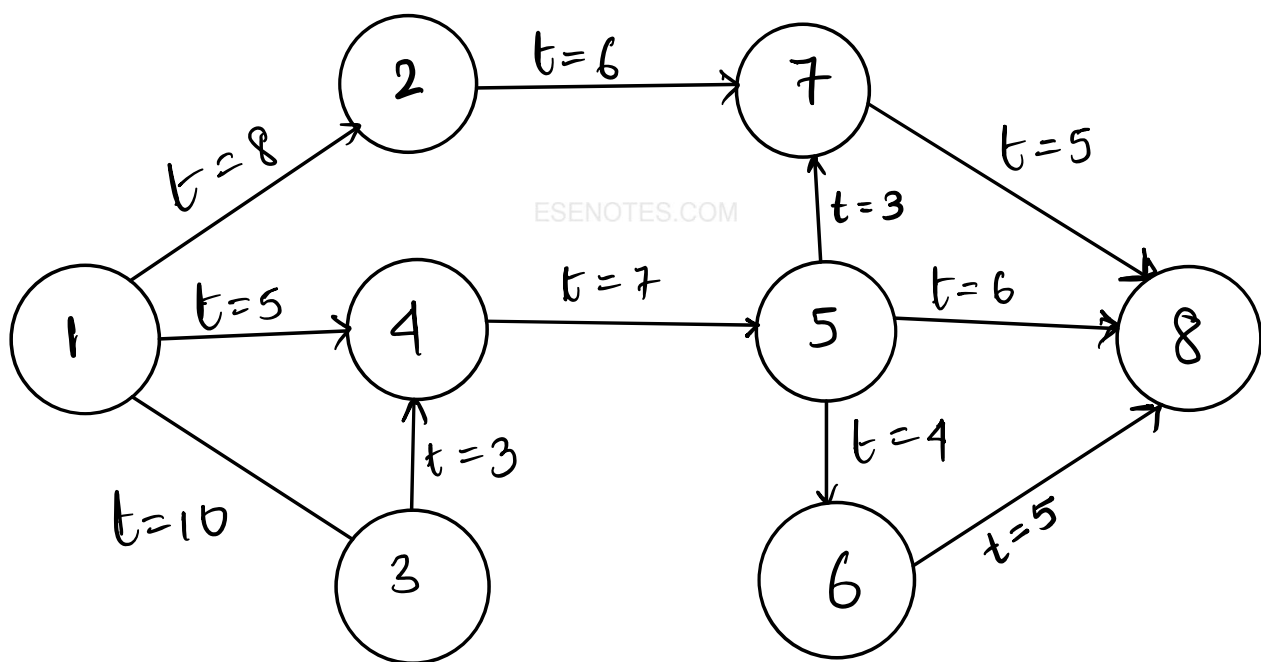


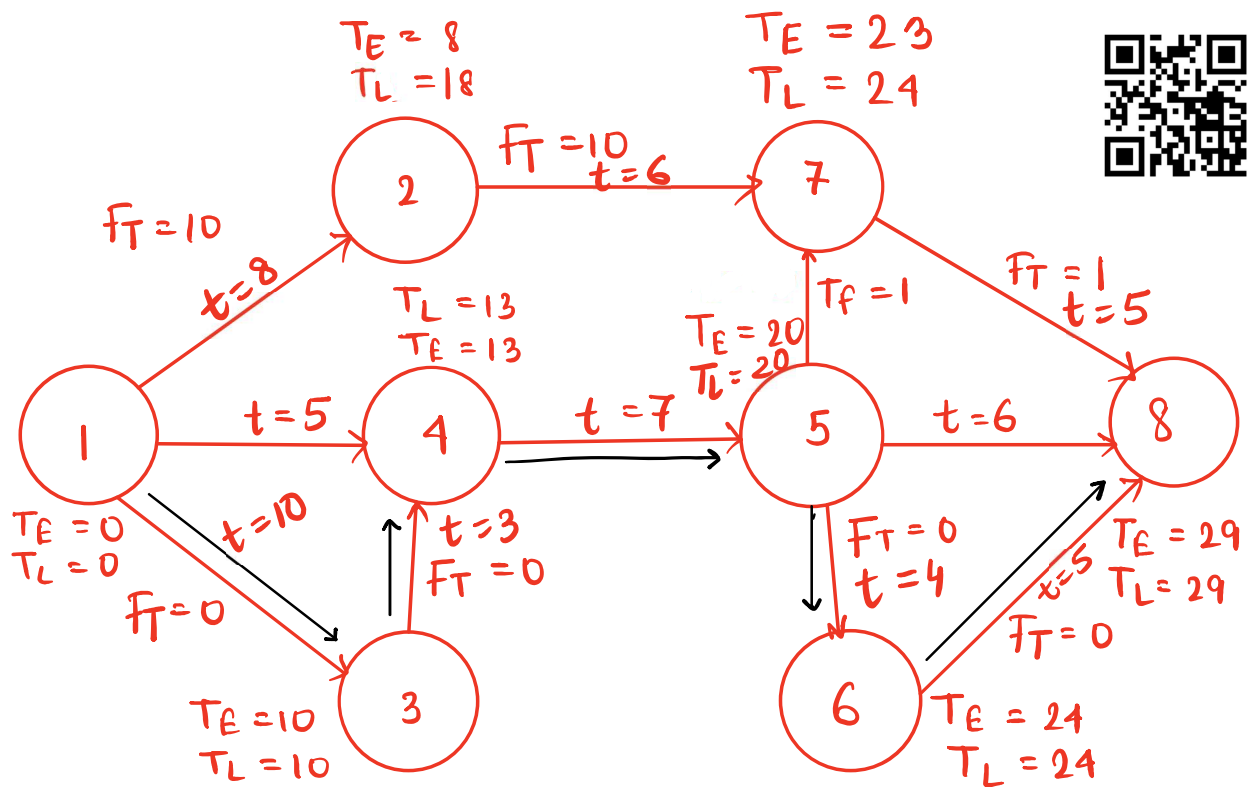
Question: fig. shows the network of project which is to be updated at the end of 12 days the following condition exist at time of updating.

- a) Activity 1-4 was completed as originally planned.
- b) Activity 1-3 was executed more rapidly than originally schedule and took 8 days for its completion.
- c) Activity 3-4 commenced following the completion of Activity 1-3 and was finished at the end of 11th day.
- d) Activity 4-5 was commenced following the completion of activity 3-4 and still required 6 more day.
- e) Completion of activity 1-2 was delayed drastically and it still requires to more 10 days for its completion.
- f) Activity 2-7 will commence following completion of activity 1-2 and will require 9 days for it completion.

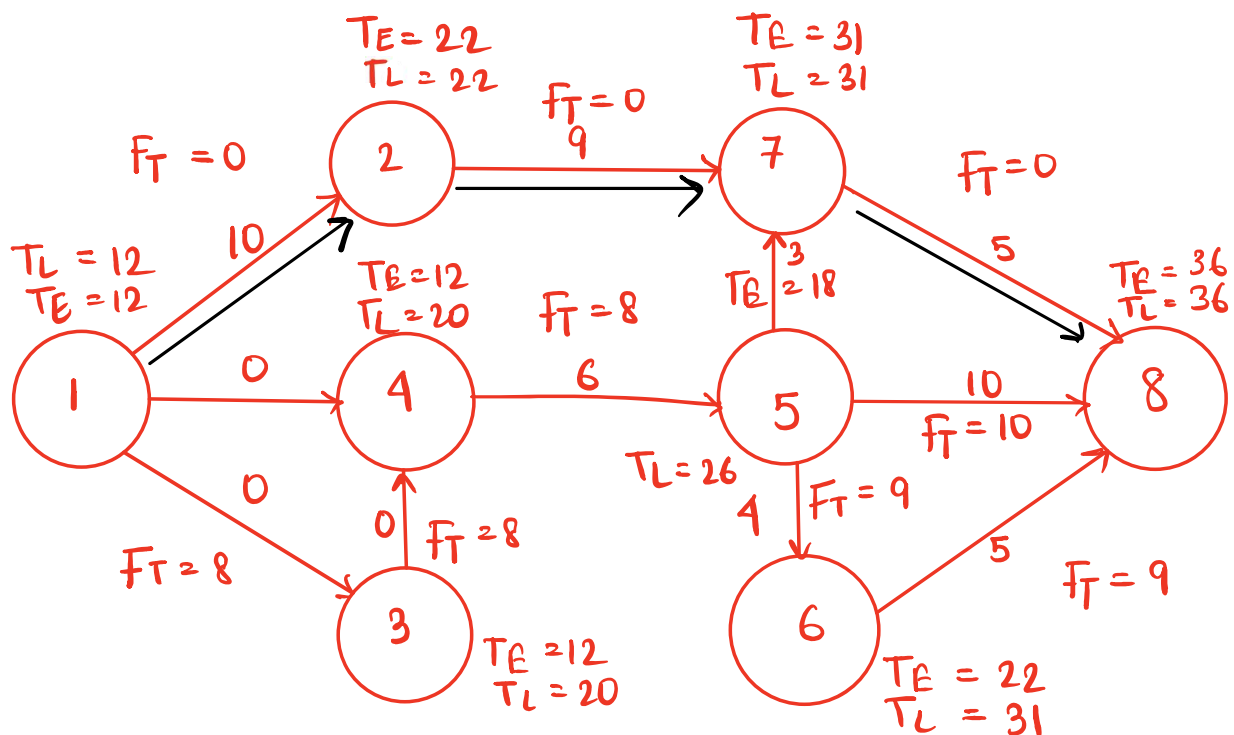
g) The time required to perform activity 5-8 has been revised to 10 days instead of 6 days.

h) No other activity have been started and original time of these activity still hold good. Update the network and time revised critical path.

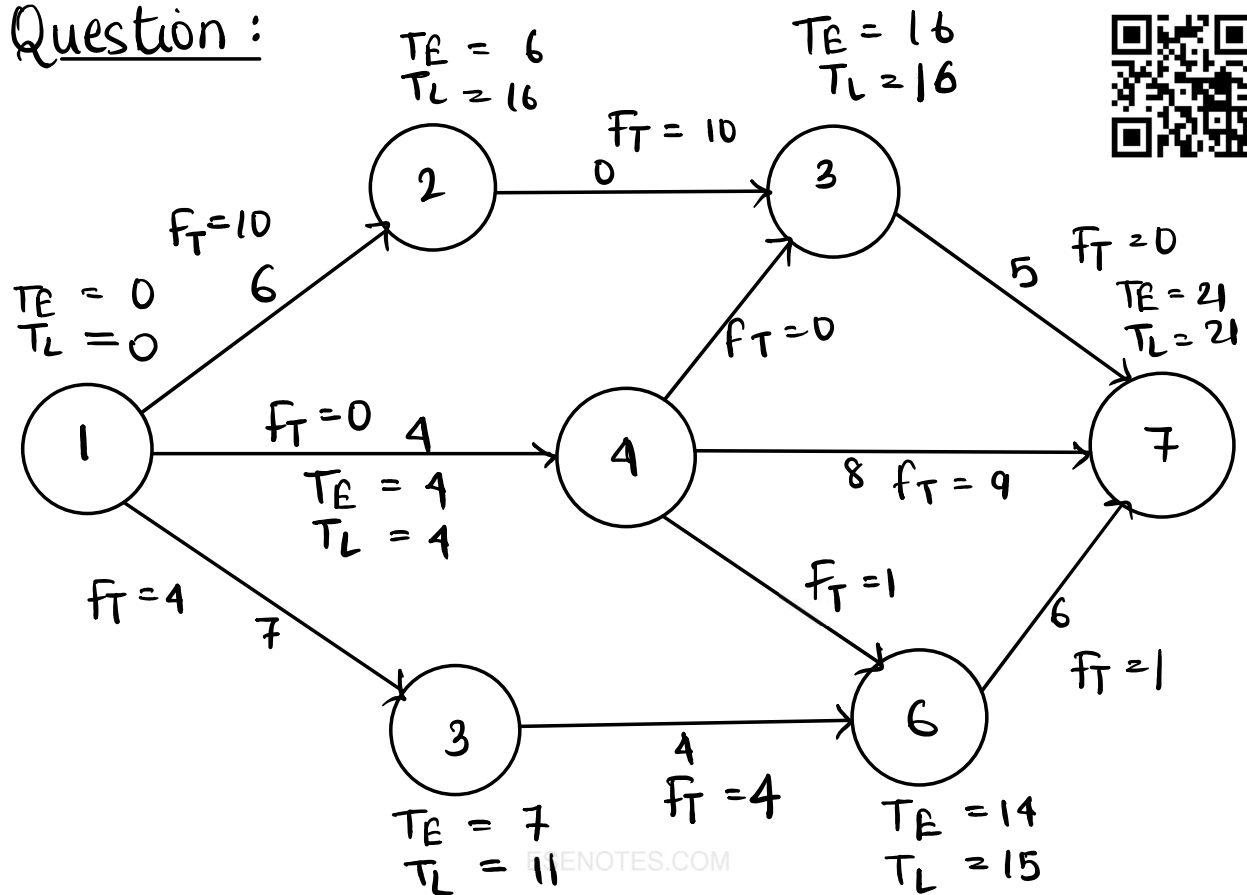




Critical path 1-3-4-5-6-8
 New Critical path 1-2-7-8.



Question :



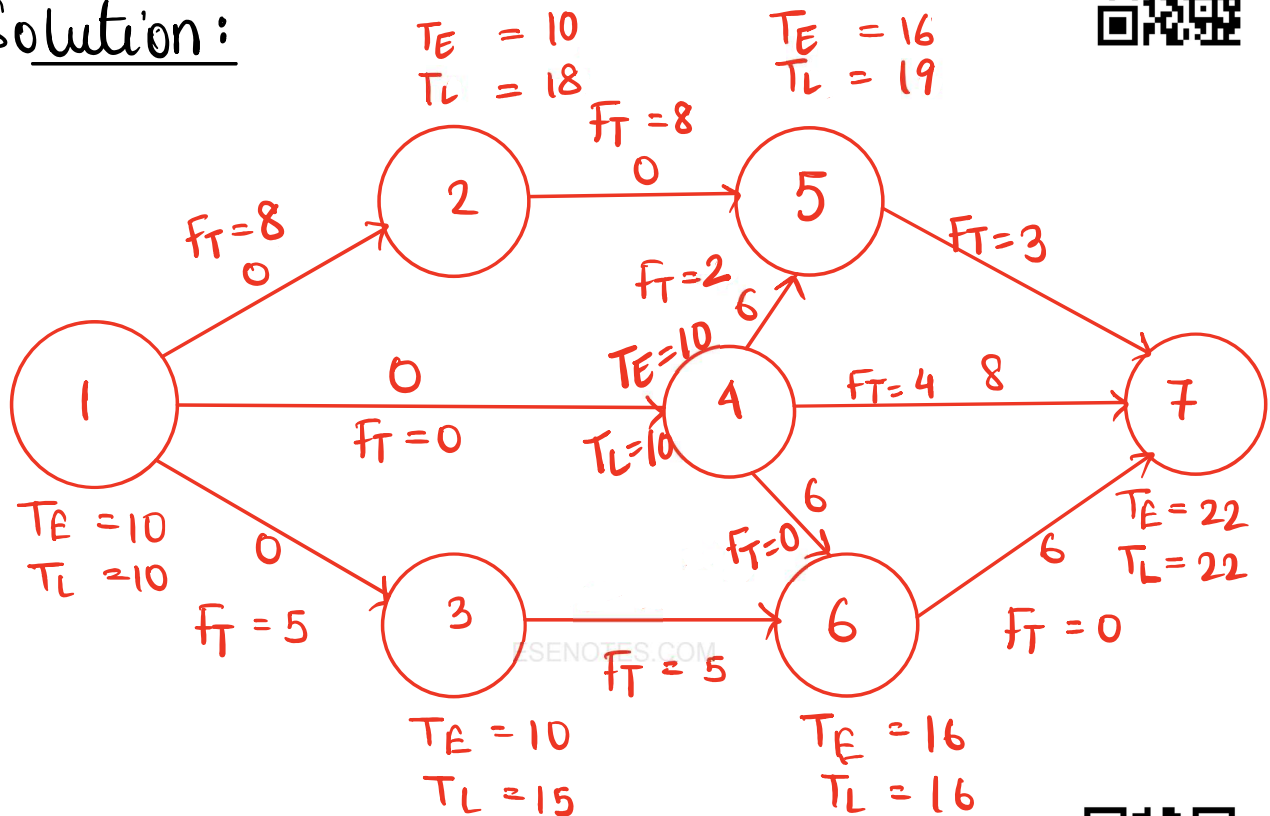
Update network after 10 days

1. Activity 1-2, 1-3, 1-4, 1-4 have been completed.
2. Activity 4-5 running still required 6 more days.
3. Activity 4-6 running still required 6 more days.
4. 3-6, 4-6 running still required 1 more days.
5. Other activity have not commenced and original predicted duration is right

except activity 5-7. Activity 5-7 required 3 days if required 5 days.



Solution:



Critical path 1-4-6-7



Resource Allocation: Resource is a physical variable required for completion of an activity or project.

Ex: Man power, machine, material, time

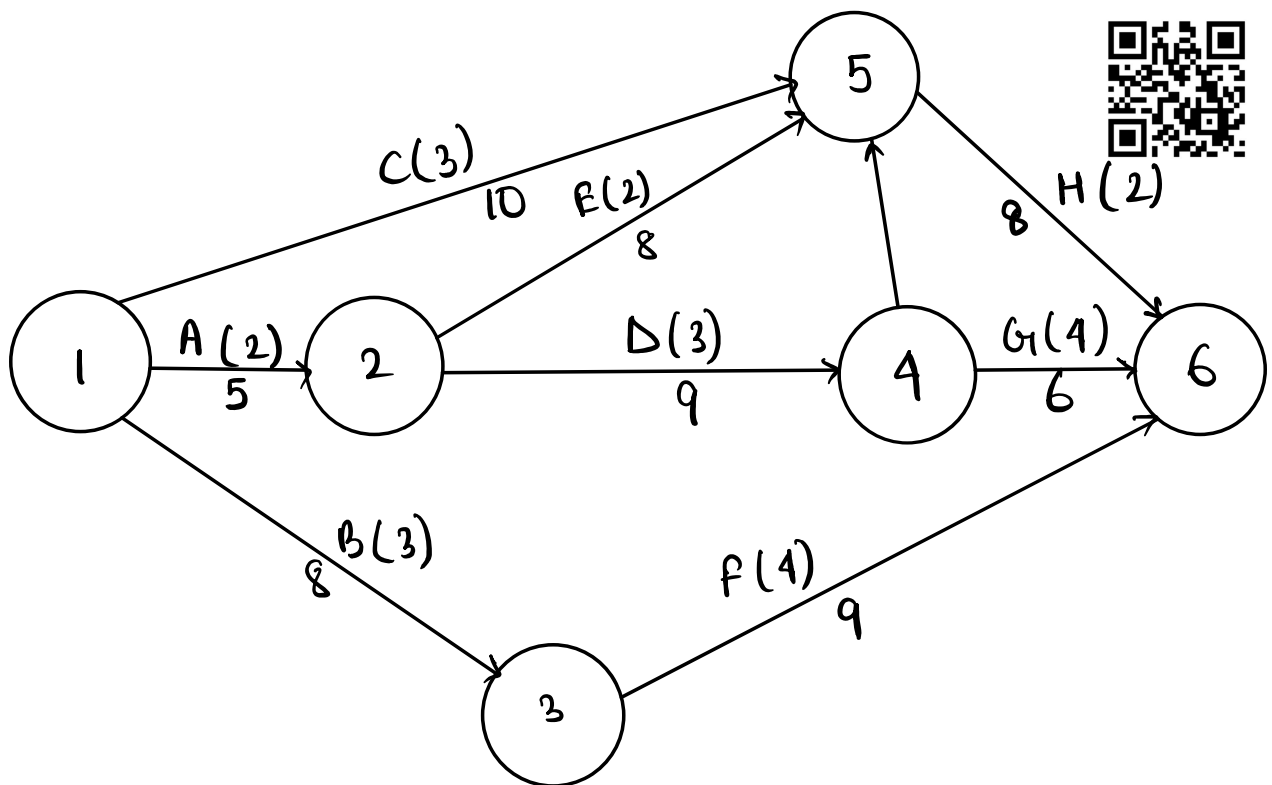
- Allocation of these resources to the activities is done during scheduling phase only by any of the following method.

a) Resource Smoothing : In this case demand of the resources is made more or less uniform throughout the duration of project. It is achieved by delaying starting or finishing of the activities by their available float.

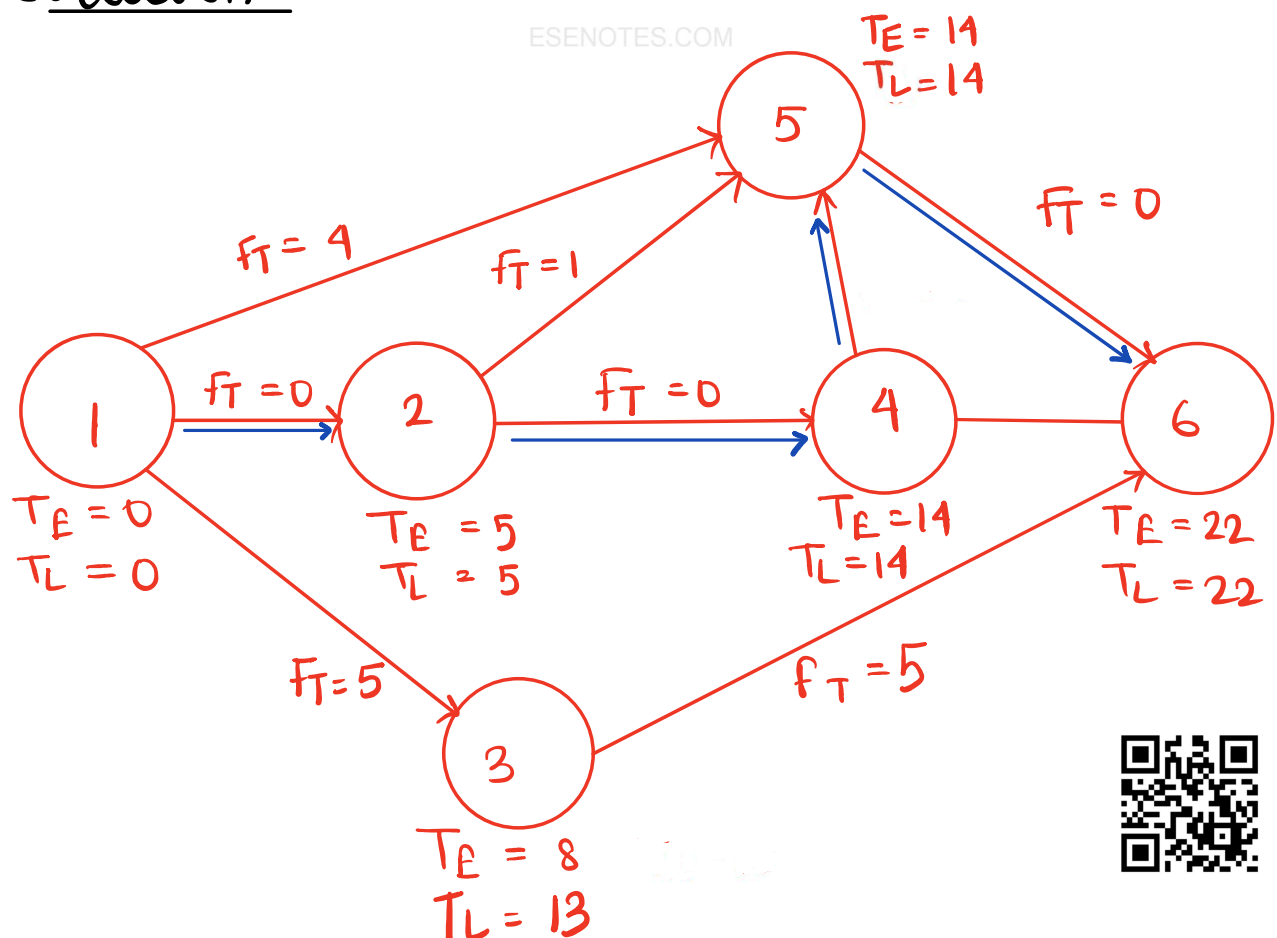
- In this case critical activity, critical path, project duration does not change.
- Here availability of resources is considered to be limited.



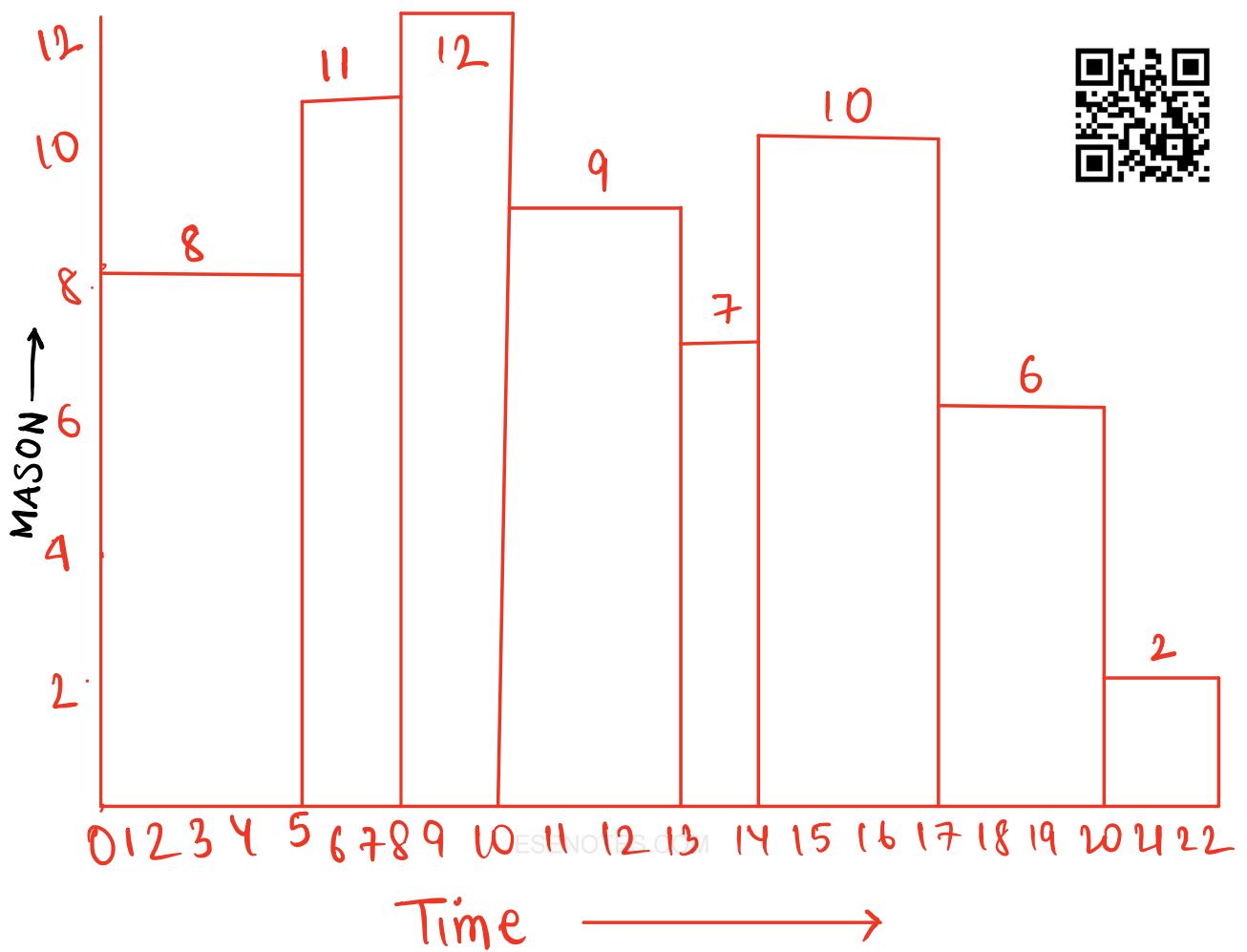
Question 1 : Determine the aggregate resource requirement period by period for the network given below. The fig. over the arrow indicate requirement of manson and fig below the arrow indicate the duration of the activities for the project.



Solution:



Time(days)	Activity	Mason
1	A, B, C	8
2	A, B, C	8
3	A, B, C	8
4	A, B, C	8
5	A, B, C	8
6	B, C, D, E	11
7	B, C, D, E	11
8	B, C, D, E	11
9	C, D, E, F	12
10	C, D, E, F	12
11	D, E, F	9
12	D, E, F	9
13	D, E, F	9
14	D, F	7
15	F, G, H	10
16	F, G, H	10
17	F, G, H	10
18	G, H	6
19	G, H	6
20	G, H	6
21	H	2
22	H	2

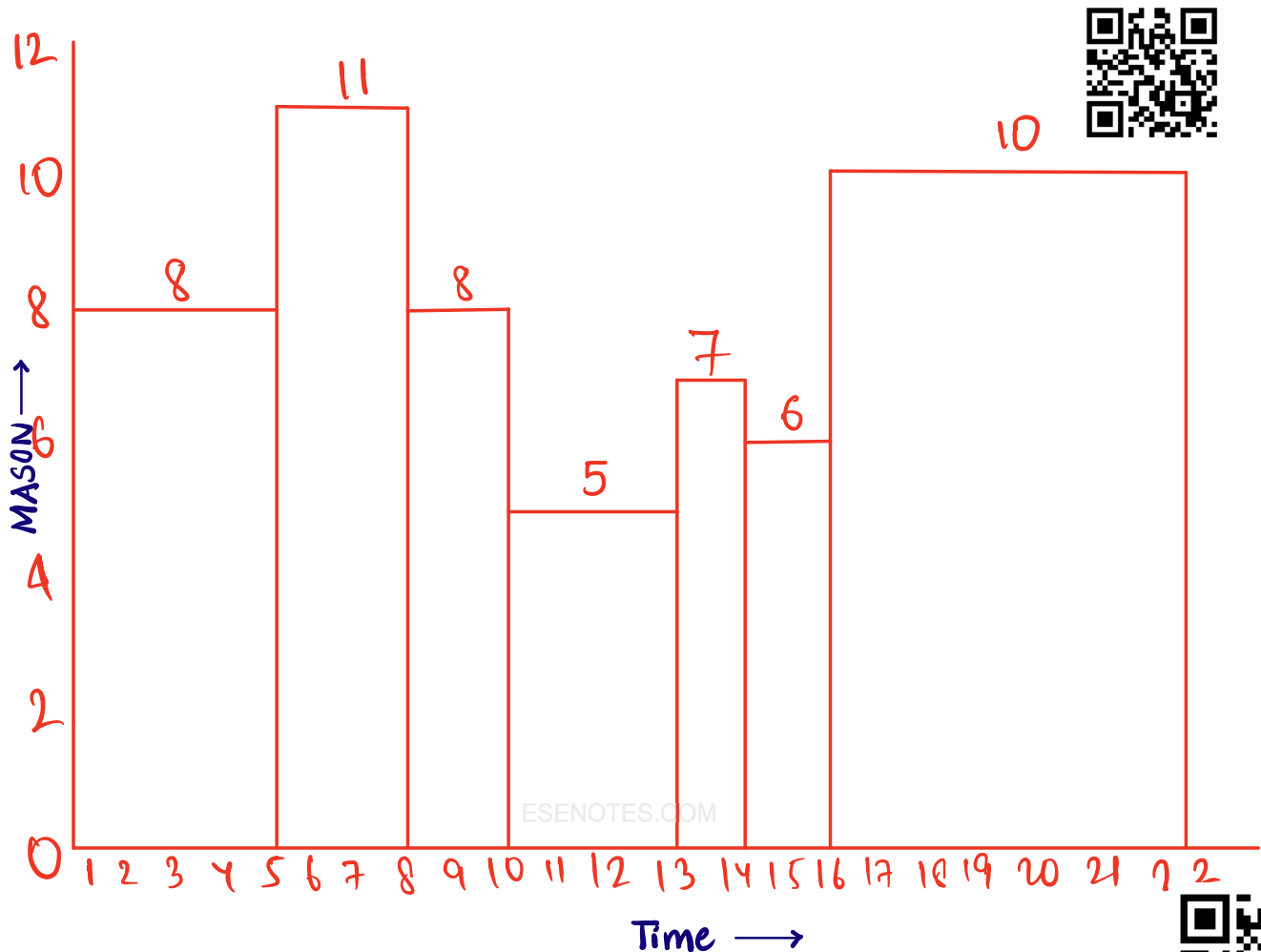


In between 6-17 Resource requirement is high.
 B C ~~D~~ E F G H
 f → delay by 5 days
 G → delay by 2 days.

Time	Activity	Mason
1	A, B, C	8
2	A, B, C	8
3	A, B, C	8
4	A, B, C	8

5	A, B, C	8
6	B, C, D, E	11
7	B, C, D, E	11
8	B, C, D, E	11
9	C, D, E	8
10	C, D, E	8
11	D, E	5
12	D, E	5
13	D, E	5
14	D, F	7
15	F, H	6
16	F, H	6
17	ESENOTES.COM F, G, H	10
18	F, G, H	10
19	F, G, H	10
20	F, G, H	10
21	F, G, H	10
22	F, G, H	10



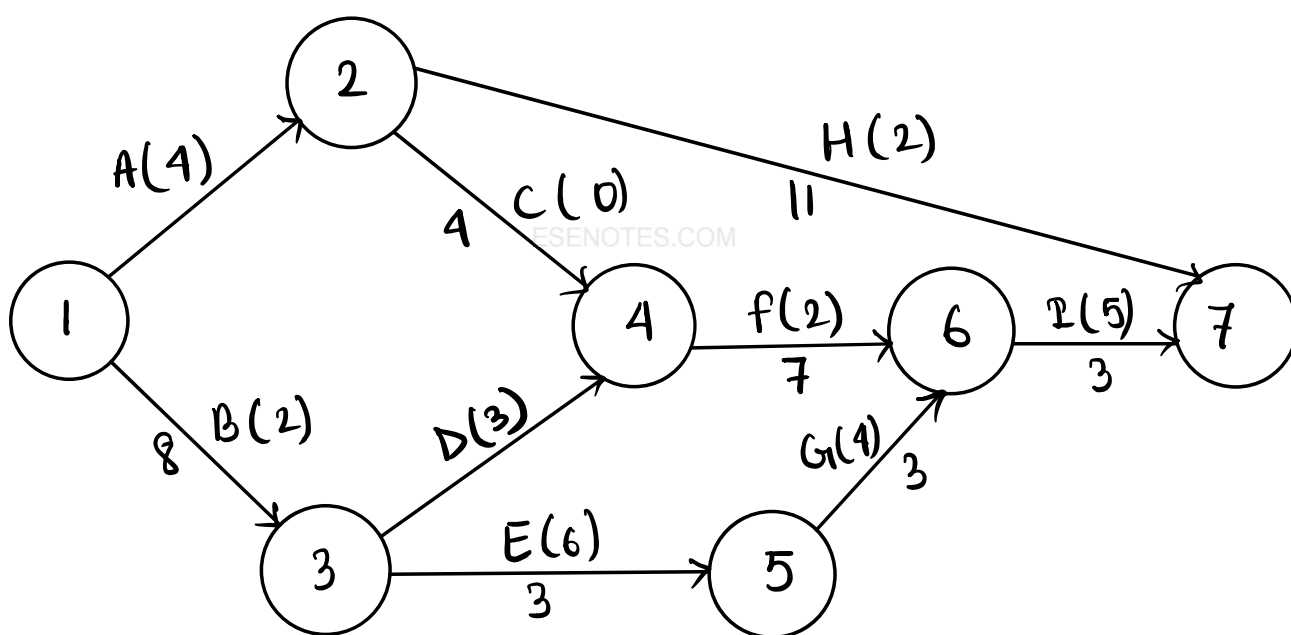


- (b) Resource levelling: In resource levelling activities are rescheduled in such a way that the maximum demand of the resources become less than the availability of resources.
- In this case, critical activities, critical path project duration may get changed during levelling.



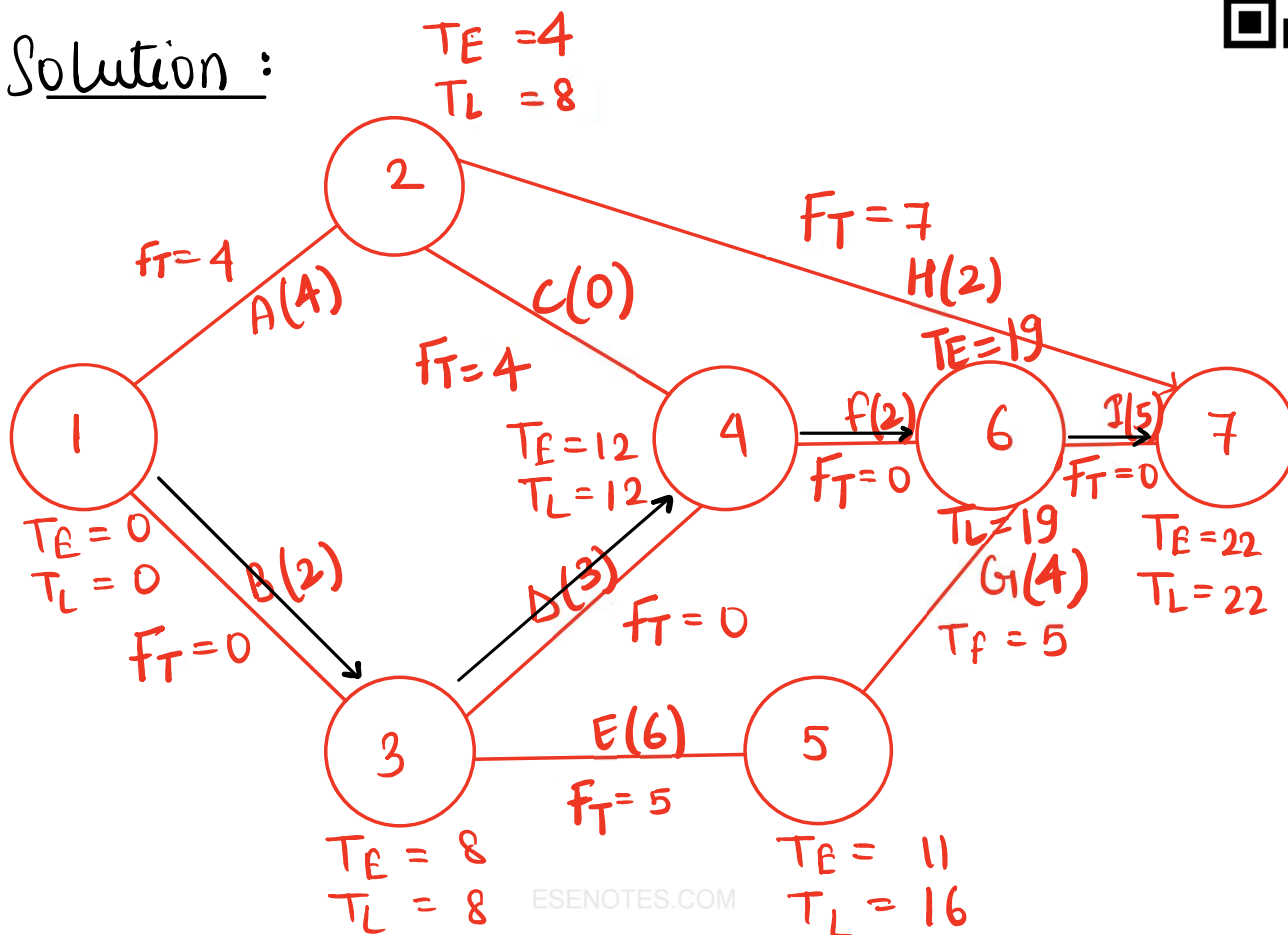
Question: Consider the network shown below the requirement of labour for each of the activities has been shown in bracket over the arrow and time duration below the arrow.

- Level out the requirement of resource if the maximum number of labours on any day available are 7 only.





Solution :

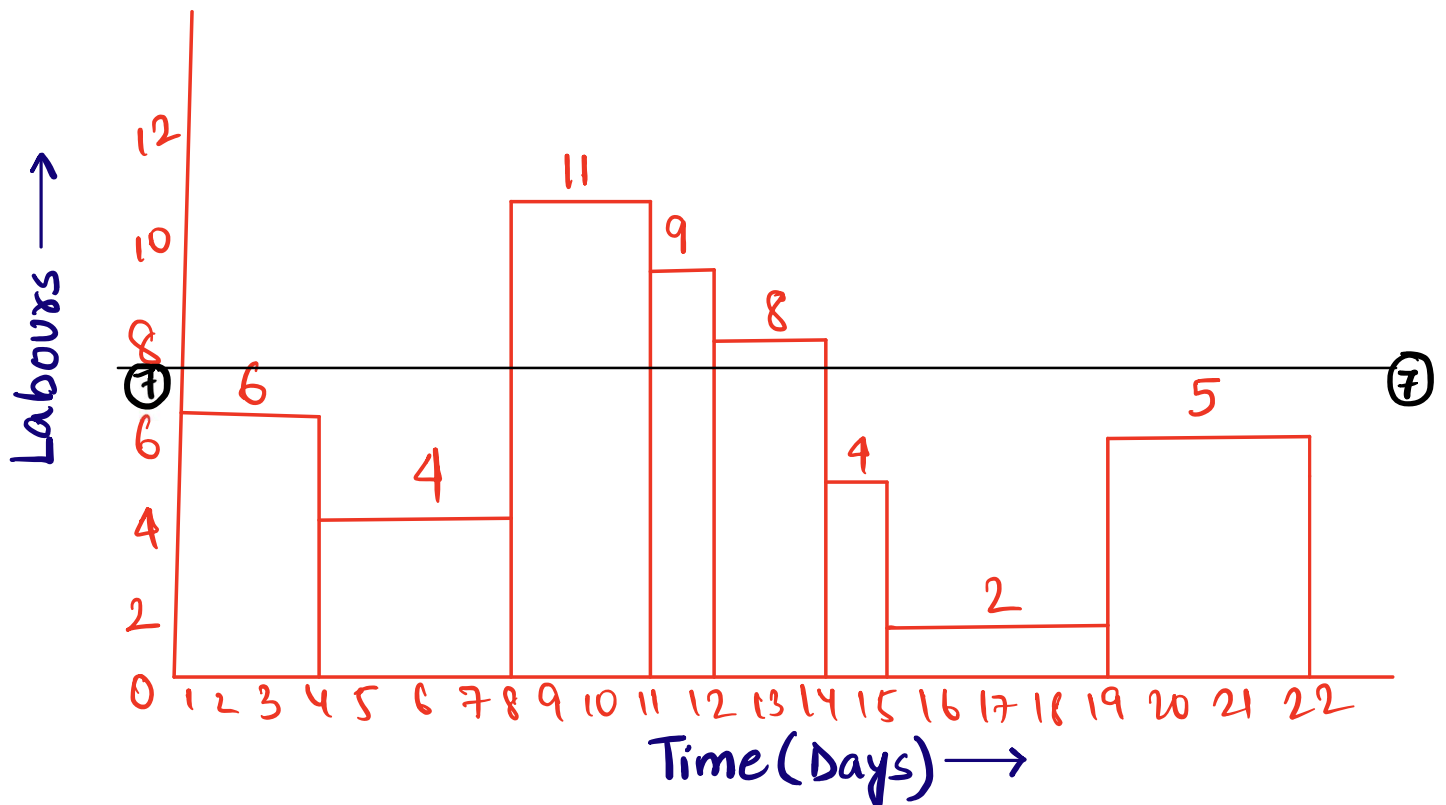


Critical Path 1-3-4-6-7.

Time	Activity	Resources
1	A, B	6
2	A, B	6
3	A, B	6
4	A, B	6
5	B, H, C	4
6	B, H, C	4
7	B, H, C	4
8	B, H, C	4



9	D, H, E	11
10	D, H, E	11
11	D, H, E	11
12	D, H, G	9
13	H, f, G	8
14	H, f, G	8
15	f, H	4
16	f	2
17	f	2
18	f	2
19	f	2
20	1	5
21	1	5
22	1	5





Activities in between 8-14

H, ~~f~~, G, ~~D~~, E
E G H

Delay activity by 5 day E and G by 5 days.

Time	Activity	Resources
1	A,B	6
2	A,B	6
3	A,B	6
4	A,B	6
5	B, C, H	4
6	B, C, H	4
7	B, C, H	4
8	B, C, H	4
9	D, H	5
10	D, H	5
11	D, H	5
12	D, H	5
13	f, H	4
14	f, E, H	10
15	f, E, H	10
16	f, E	8
17	f, G	6



18	f, G	6
19	f, G	6
20	I	5
21	I	5
22	I	5

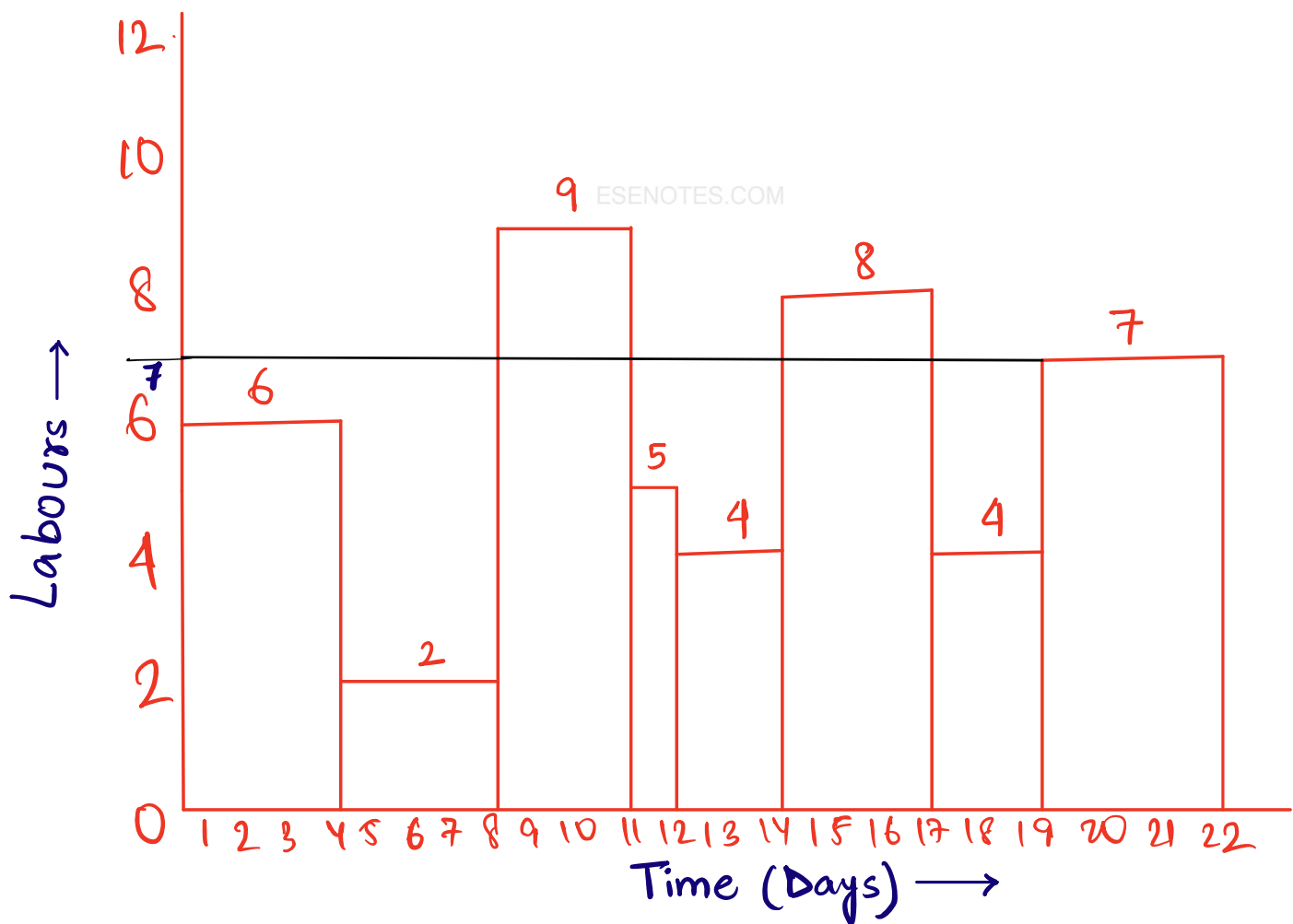
No advantages drive by delaying activity E and G.

So now delay activity H by 7 day and G by 3 day

Time	Activity	Resources.
1	A, B	6
2	A, B	6
3	A, B	6
4	B, A	6
5	B, C	2
6	B, C	2
7	B, C	2
8	B, C	2
9	D, E	9
10	D, E	9
11	D, E	9
12	D, H	5
13	H, F	4



14	H, f	4
15	f, H, G	8
16	f, H, G	8
17	f, H, G	8
18	f, H	4
19	f, H	4
20	I	7
21	I, H	7
22	I, H	7





9-17 duration of no critical activity increase by doing of demand decrease.

9-17 ~~D~~ E ~~F~~ G ~~H~~ Already delayed.

Activity duration of E can be increased by 5 day.

Already delayed by 2.

E $\longrightarrow$
 3 day $\longleftarrow$ 6 labour
 6 day $\longleftarrow$ 3 labour
 G $\longrightarrow$
 3 day $\longleftarrow$ 4 labour
 4 day $\longleftarrow$ 3 labour.

} duration of E $\longrightarrow$ 6 day
 } Duration of G $\longrightarrow$ 4 day

Critical Activity O
 4 days $\longleftarrow$ 3 labour
 6 day $\longleftarrow$ 2 labour

} Duration of activity D $\longrightarrow$ 6 day

Time	Activity	Resources
1	A , B	6
2	A , B	6



3	A , B	6
4	B , C	6
5	B , C	2
6	B , C	2
7	B , C	2
8	D , E	2
9	D , E	6
10	D , E	6
11	D , E	6
12	D , H , E	8
13	H , f , E , D*	7
14	H , f , E , D*	7
15	f , H , G	7
16	f , H , G	7
17	f , H , G	7
18	f , H , G	4
19	f , H	4
20	1 H	7
21	1 H	7
22	1 H	7.

PREVIOUS YEAR QUESTION

ESE

~~Question 1: Statement I: Resource~~

— optimization is largely a pre-implementation pursuit whereas resources allocation is a through implementation dynamic process.

Statement II: Resource allocation has a larger bearing on inventory management than resource optimization.
(2016).

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Solutions: c)

Resource allocation has effect on inventory management as well as resource optimization because availability of resource will also decide project completion time.

Question 2: Resource smoothing is

- a) An optimization and economical utilization of resource.
- b) An adjustment of resource without affecting project duration.
- c) A gradual increase in resources

d) A gradual decrease in resources.
(2020)

Solution: b)

In this total project duration is not changed but some of the activity start times are shifted by their available float so that more or less uniform demand is generated.

Question 3: What is the process of incorporating changes and rescheduling or replanning called

- a) Resource allocation
- b) Resource levelling
- c) Resource smoothing.
- d) Updating.

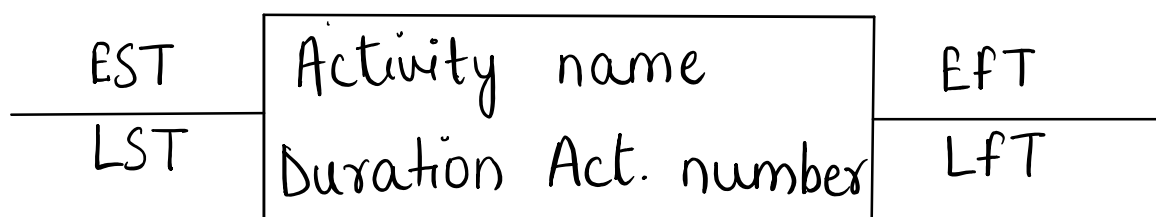
(2020)

Solution: d) Updating.



AON : Activity on node network diagram or Precedence diagram.

- In these network diagram, activities are marked nodes and there is no place for event.
- Here arrows indicates the relationship between the activities, hence it is also termed as precedence / Interdependence diagram.
- Here there is no requirement of dummy, hence it is simpler to analyse.
- This diagram is self explanatory / self sufficient.
- Here representation of activities on node is done as follows.



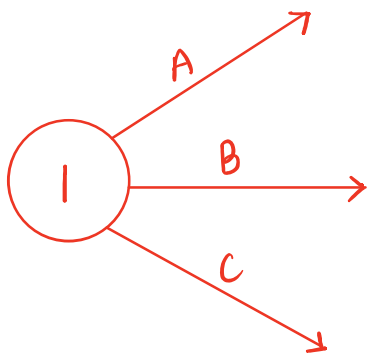


NOTE: If more than one activity starts from initial event in AOA network then a DEBUT activity is used in AON network.

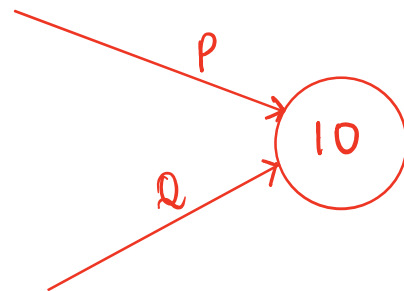
Similarly, If more than one activity terminates at finish event in AOA network then FINISH activity is used in AON network.

for eg :

AOA

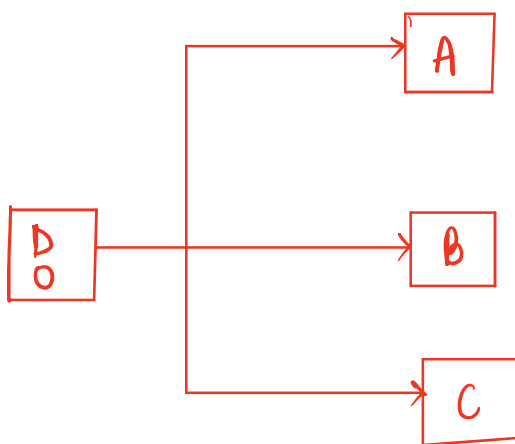


AOA



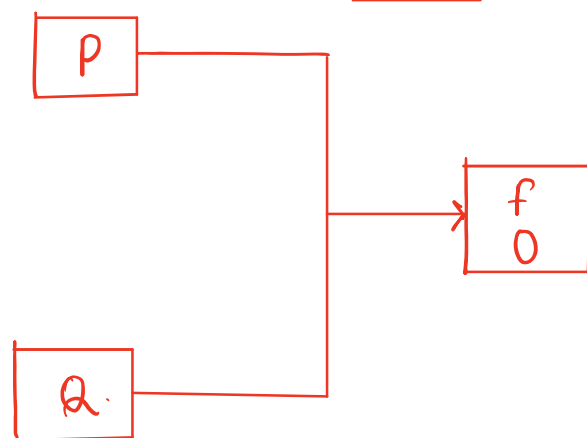
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AON



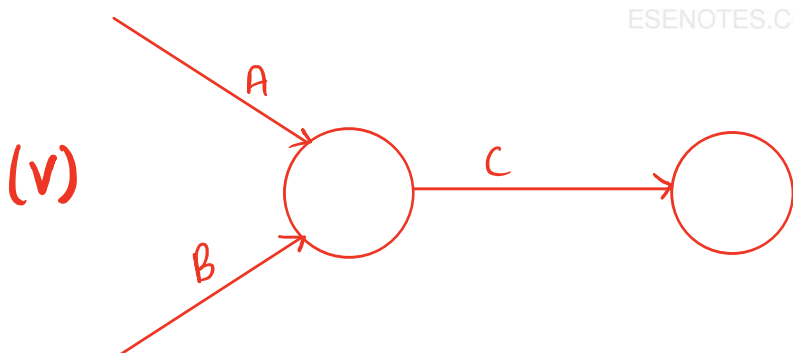
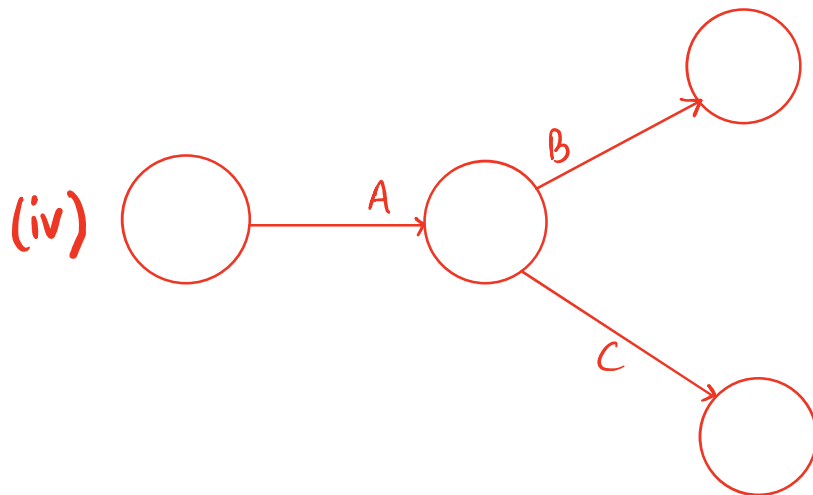
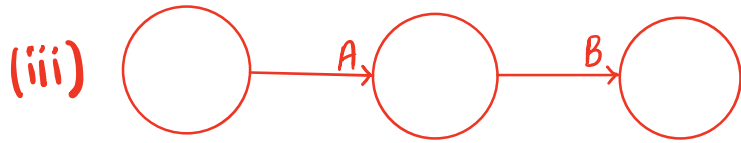
(i)

AON

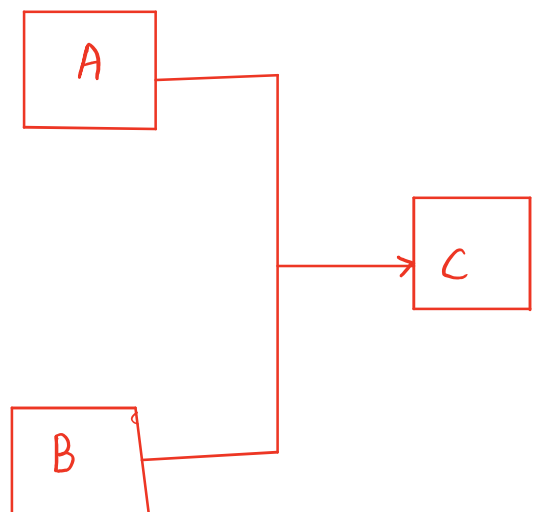
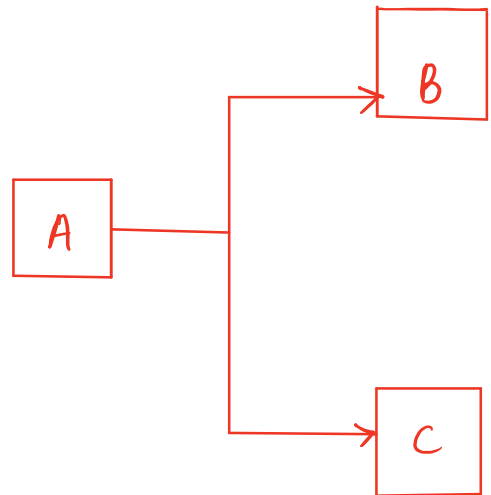


(ii)

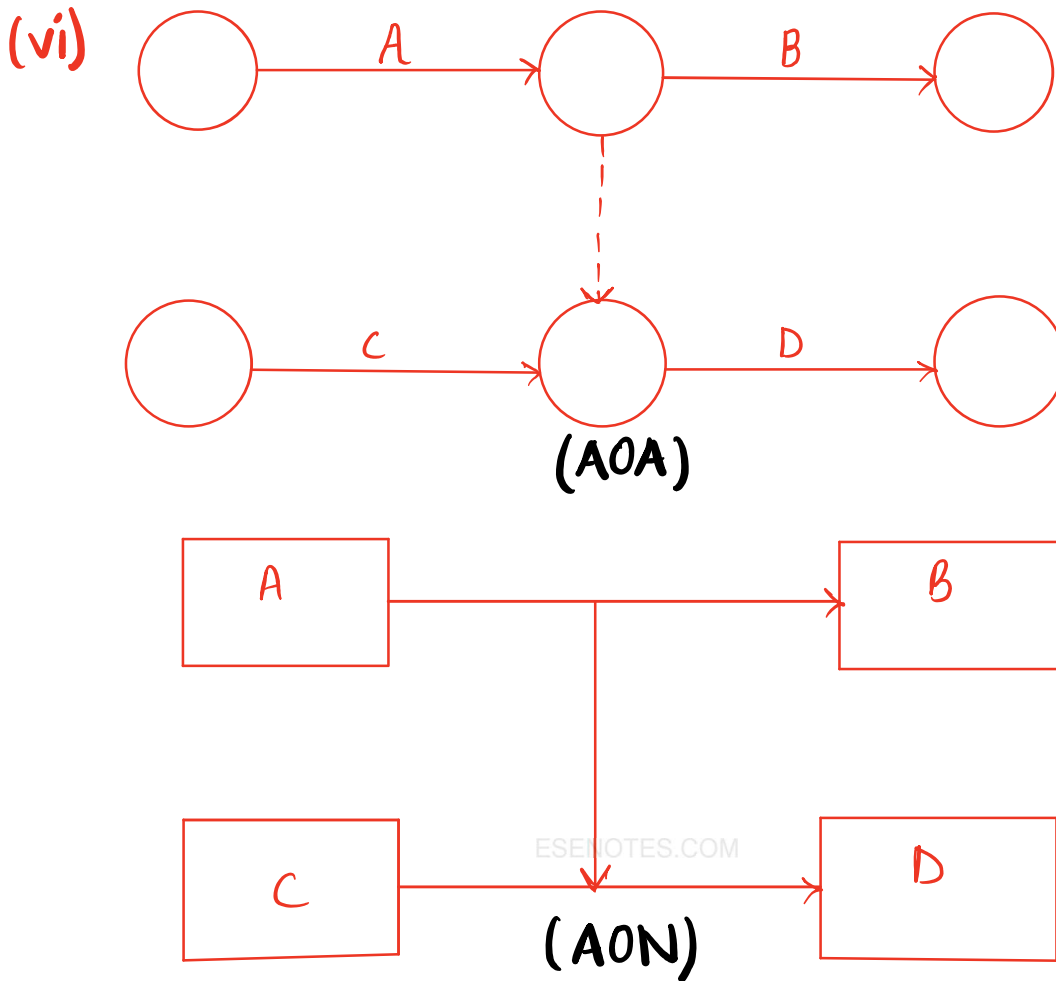
AOA



AON

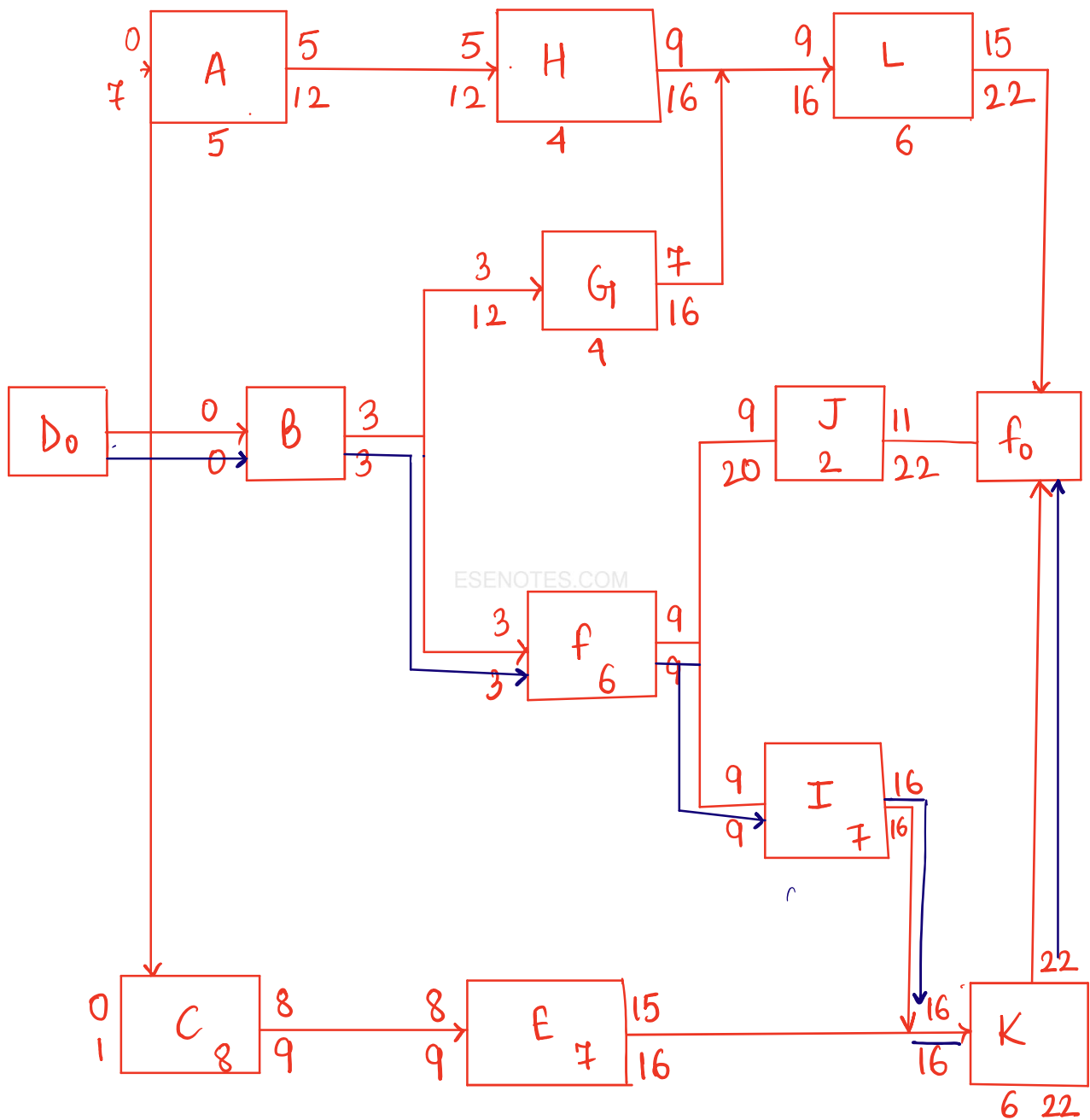


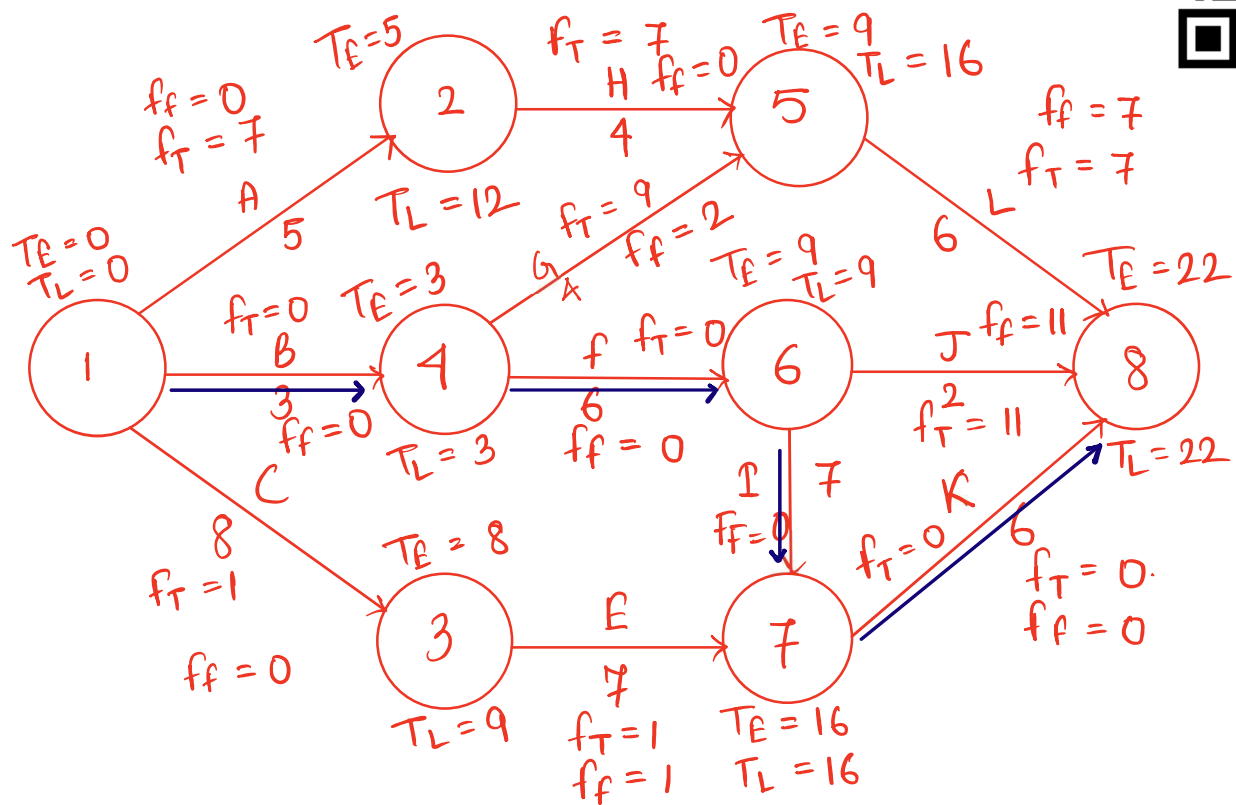
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Question: The network of a certain project is shown in fig. with the estimated durations of various activities. Draw the corresponding AON network and determine

- Earliest and latest activity time.
- Total and free float of activities.
- Critical Path.





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Question : Three act f, G, H are to be performed in said sequence and for the whole stretch of the project needs 15, 21, 27 days respectively. If a ladder network is adopted with one third of each activity as the ladder activity, the saving in total project time would be

- a) 15
c) 21

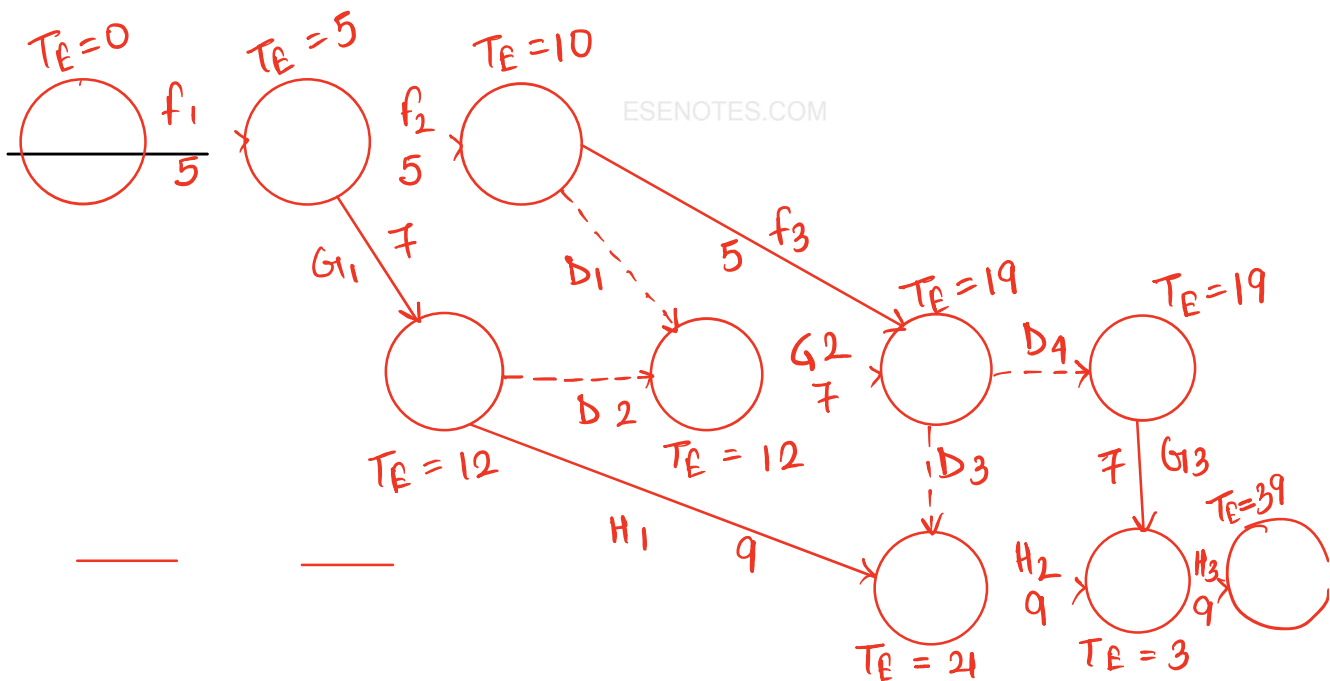
- b) 24
d) 18

$$(1) \begin{matrix} F \\ 15 \end{matrix} \times (2) \begin{matrix} G \\ 21 \end{matrix} \times (3) \begin{matrix} H \\ 27 \end{matrix} \times (4) \quad T_E = 63$$



Solution :

I	II	III
f_1	f_2	f_3
G_1	G_2	G_3
H_1	H_2	H_3



PREVIOUS YEAR QUESTION

ESE

Question 1: Which one of the following is relevant to activity on Node (AON) ?

- a) Dummy activities may be many.
- b) There will be no dummy activities.
- c) It is used for quite complex project.
- d) It is easier to associate with time flow of activities.

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(2010)

Solutions : b)

• points about A-O-N diagram -

1. Events has no place, hence this system is not suitable for PERT.
2. Nodes represents activities.
3. Interdependencies is shown by arrow.
4. Dummies have no place.
5. AON diagram contains all activity time i.e, EST, LST, EFT and LFT.

Question 2: Which system of network given below completely eliminates the use of dummy activities?

- a) A-O-A (Activity-on-Arrow)
- b) A-O-N (Activity-on-Node)
- c) PERT
- d) CPM.

(2014)

Solution: b)

Question 3: Statement I: AON networks do not generally have any dummy links.

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Statement II: Links express activity dependence exhaustively and completely.

- a) both A and R are true and R is the correct explanation of A.
- b) both A and R are true but R is not a correct explanation of A.
- c) A is true but R is false
- d) A is false but R is true.

(2015)

Solution: a).